A Characterization of 1-greedy bases

by

Fernando Albiac [1]

Let X be a (real) Banach space with a semi-normalized basis $(e_n)_{n=1}^{\eta}$ (η finite or infinite). For each $m = 1, 2, \cdots$ we let Σ_m denote the collection of all elements of X which can be expressed as a linear combination of m elements of (e_n), that is, $\Sigma_m = \big\{y = \sum_{j \in B} \alpha_j e_j : B \subset \mathbb{N},\ |B| = m,\ \alpha_j' \text{s scalars}\big\}$. For $x \in X$, its *best m-term approximation error* (with respect to the given basis) is $\sigma_m(x) = \inf\limits_{y_m \in \Sigma_m} \|x - y\|$. The fundamental question is how to construct an algorithm which for every $x \in X$ and each m produces an element in Σ_m so that the error of the approximation of x by $T_m(x)$ be (uniformly) comparable with $\sigma_m(x)$, i.e. $\|x - T_m(x)\| \leq C\sigma_m(x)$ where C is an absolute constant.

The most obvious and in some sense natural attempt to get such an algorithm is to consider the *Greedy Algorithm*, $(\mathcal{G}_m)_{m=1}^{\eta}$, where for each x, $\mathcal{G}_m(x)$ is obtained by taking the largest m coefficients in the series expansion of x. To be precise, if we let $(e_n^*)_{n=1}^{\eta} \subset X^*$ denote the biorthogonal functionals associated to $(e_n)_{n=1}^{\eta}$, for $x \in X$ put

$$\mathcal{G}_m(x) = \sum_{j \in B} e_j^*(x) e_j,$$

where the set $B \subset \mathbb{N}$ is chosen in such a way that $|B| = m$ and $|e_j^*(x)| \geq |e_k^*(x)|$ whenever $j \in B$ and $k \notin B$.

Konyagin and Temlyakov defined a basis to be *greedy* if $\mathcal{G}_m(x)$ is essentially the best m-term approximation to x using the basis vectors; that is, if there is a constant $C \geq 1$ such that for all $x \in X$ and $m \in \mathbb{N}$, we have

$$\|x - \mathcal{G}_m(x)\| \leq C\sigma_m(x).$$

The smallest such constant C will be called the *greedy constant* of (e_n) and, in this case, we will say that (e_n) is C-greedy. They also defined a basis $(e_n)_{n=1}^{\eta}$ to be *democratic* if there is a constant $\Delta > 0$ such that for any two finite subsets A, B of $\mathbb{N}$ with $|A| = |B|$ we have

$$\Big\| \sum_{k \in A} e_k \Big\| \leq \Delta \Big\| \sum_{k \in B} e_k \Big\|,$$

and gave the following characterization of greedy bases:

[1] Joint work with P. Wojtaszczyk

Theorem 1 . *If $(e_n)_{n=1}^{\eta}$ is a greedy basis with greedy constant $\leq C$, then (e_n) is unconditional with unconditional basis constant $\leq C$ and democratic with democratic constant $\leq C$. Conversely, if (e_n) is unconditional with unconditional basis constant K_{ub} and Δ-democratic, then (e_n) is greedy with greedy constant $\leq K_{ub} + K_{ub}^3\Delta$.*

If we disregard constants, the previous theorem says that a basis is greedy if and only ifit is unconditional and democratic. In particular, Theorem 1 immediately yields that a 1-greedy basis has both K_{ub} and democracy constant equal to 1. However, this is not a characterization of bases with greedy constant 1.

On the oter hand, it is well known that a greedy basis in a Banach space is not necessarily a symmetric basis. For instance, the Haar system is a greedy basis for $L_p(0,1)$, $1 < p < \infty$, which is not symmetric unless $p = 2$. Thus the greedy bases are somewhere in between of unconditional bases and symmetric bases.

In this talk we investigate with a little bit more of detail how greedy bases which have greedy constant $= 1$ relate to symmetric basis and give an answer to the problem of finding a characterization of 1-greediness.

On the topological invertible elements of a topological algebra

by

Hugo Arizmendi-Peimbert

Let A be a topological algebra with unit e, we say that $a \in A$ is *topologically invertible* if $\overline{aA} = \overline{Aa} = A$. In such a case there exists a pair of nets $\widetilde{a} = (a_\lambda)$ and $\widetilde{b} = (b_\lambda)$, called *right* and *left topological inverses* respectively, such that $aa_\lambda \to e$ and $b_\lambda a \to e$. Conversely, a net $\widetilde{a} = (a_\lambda)$ in A is called *advertible convergent* (shortly *advertible*) if there exists $a \in A$ such that $aa_\lambda \to e$ and $a_\lambda a \to e$. Thus, if (a_λ) is convergent, then $a_\lambda \to a^{-1}$. A topological algebra is called *advertibly complete* if every Cauchy advertible net is convergent.

Bhatt and Thate have shown that in topological Q-algebras as well as in complete locally multiplicatively convex algebras every invertible element is, in fact, invertible. In this talk we study the relation between bounded topological invertibility and invertibility, according to the definition of a *bounded* *net* given by H. Arizmendi and R. Harte and called by W. Zelazko *almost bounded* and *ultimately bounded* by R. Vera.

Iterative procedures for solutions of random operator equations in Banach spaces

by

Ismat Beg

Abstract ___ We construct random iterative processes for weakly contractive and asymptotically nonexpansive random operators and study necessary conditions for the convergence of these processes. It is shown that they converge to the random fixed points of these operators in the setting of Banach spaces. We also proved that an implicit random iterative process converges to the common random fixed point of a finite family of asymptotically quasi nonexpansive random operators in uniformly convex Banach spaces.

Keywords and Phrases: Random Mann iteration, random Ishikawa iteration, random operator, three step random iterative process, implicit random iterative process, Banach spaces, measurable spaces.

Projective description for weighted inductive limits of spaces of holomorphic functions

by

Klaus D. Bierstedt

It was a problem since Ehrenpreis' work on "analytically uniform spaces" whether the topology of a countable inductive limit of weighted spaces of holomorphic functions can again be described by weighted sup-seminorms. This was also known as the problem whether there is a *projective description* for the inductive limit. A general positive answer in the case of (LB)-spaces, sufficient for many applications, was already given in a joint article of the author with R. Meise and W.H. Summers published in Trans. AMS in 1982. Counterexamples to projective description for (LB)-spaces of holomorphic functions were constructed only much later, in 1995-2000, by J. Bonet, S. Melichov, J. Taskinen, and D. Vogt. In the first part of this talk, we will survey the known positive results, the counterexamples and some interesting open questions.

In the second part, we will present recent results from two joint articles with J. Bonet, one of which appeared in Proc. Edinburgh Math. Soc. in 2003; the other one is still in preparation. The first of these articles also dealt with the more general case of (LF)-spaces, but the holomorphic functions had to be defined on the unit disc, and the weights had to be radial and normal in the sense of A.L. Shields and D.L. Williams. In the article under preparation we again restrict our attention to (LB)-spaces and investigate the relation of the four spaces $\mathcal{V}H(G)$, $\mathcal{V}_0H(G)$, $H\overline{V}(G)$, and $H\overline{V}_0(G)$ in the case that G is the unit disc or the complex plane. Under rather strong assumptions, we can then show that the behavior of the spaces of holomorphic functions is similar to the behavior of the corresponding sequence spaces.

Surjective convolution operators on spaces of vector valued distributions

by

José Bonet

We report on joint work with P. Domanski.

The following problem is considered: Suppose that T is a surjective convolution operator on the space of distributions $D'(G)$ on an open subset G of R^N. Let E be a complete locally convex space. Under which conditions is the vector valued convolution operator $T \otimes Id$ from $D'(G, E)$ into $D'(G, E)$ also surjective? Results of this type are motivated by the problem of solving of linear partial differential operators depending on a parameter. The most interesting cases are when E is a Fréchet Schwartz space or a (LS)-space, i.e. the dual of a Fréchet Schwartz space.

Several results were already known. Partial results for operators on spaces of vector valued real analytic functions were obtained by us. In the case of distributions, when T is a hypoelliptic linear partial differential operator $P(D)$ with constant coefficients and G is convex, $P(D) \otimes Id$ is surjective on $D'(G, E)$ for every Fréchet space E and for complete (LB)-spaces E which satisfy the condition (A) of Vogt. These results were obtained by Petszche in 1980. He had continued the work of Vogt. In fact, Vogt proved in 1983 that, for certain elliptic operators $P(D)$, the condition (A) is necessary and sufficient for the surjectivity of $P(D) \otimes Id$ if the space E is a complete (LB)-space.

The problem mentioned above is related to the theory of the vanishing of the functor Ext^1 in the category of (PLS)-spaces (i.e., projective limits of sequences of duals of Fréchet-Schwartz spaces) and with the theory of the vanishing of the first derived functor of the projective limit functor for locally convex spaces. We present characterizations of those pairs (F, X) of a Fréchet Schwartz space F (which is the dual of E in our problem) and an ultrabornological (PLS)-space $X = proj_N X_N$ (which is the kernel of T in our problem) such that every exact sequence of (PLS)-spaces

$$0 \to X \to Y \to F \to 0$$

splits. More concrete evaluations for power series spaces F or for X a sequence space, the kernel of a convolution operator on $D'(G)$ or a space of quasianalytic functions on the real line have been obtained. We are able to omit the ellipticity or hypoellipticity assumptions of Vogt and Petszche, and prove that if a convolution operator $T : D'(G) \to D'(G)$ is surjective (G convex), then the E-valued corresponding operator is surjective, when E is the dual of a power series space of infinite type (for instance, when E is isomorphic to the space of germs of holomorphic functions over a point).

A uniqueness theorem for the Korenblum space

by

Alexander Borichev

A result of Lyubarskii–Seip (1997) characterizes the (radial) weight functions h such that for every (hyperbolically) separated non-Blaschke set $E \subset \mathbb{D}$

$$f \in H^\infty, \quad |f(z)| \le h(z),\, z \in E \implies f = 0.$$

We extend this result to the Korenblum space $A^{-\infty}$ consisting of functions f holomorphic in the unit disc and such that

$$\sup_{z \in \mathbb{D}} \frac{\log |f(z)|}{\log[1/(1 - |z|)]} < \infty.$$

Complex interpolation and twisted twisted Hilbert spaces

by

Jesús M. F. Castillo

Complex interpolation tools can be understood from a homological point of view. With this moto in mind we review the ideas of Carro, Cerda and Soria [Commutators and interpolation methods, Ark. Mat. 33 (1995) 199–216] about compatible couples of interpolators and read them in the context of exact sequences of Banach spaces: interpolators become natural transformations, derived spaces become twisted sums, etc . Then, using the drawing of diagrams as a guide, we move to higher-order interpolators and show they fit in the general homological schema.

To present an application, recall that a twisted Hilbert space is a Banach space X admitting a Hilbert subspace H such that the corresponding quotient X/H is also a Hilbert space. In homological terms, the middle space in an exact sequence

$$0 \longrightarrow l_2 \longrightarrow X \longrightarrow l_2 \longrightarrow 0.$$

There exist twisted Hilbert spaces that are not Hilbert spaces, such as Kalton-Peck Z_2 space [Twisted sums of sequence spaces and the three space problem. Trans. Amer. Math. Soc. 255 (1979), 1–30]. All those who know it know that the first outcome of Schechter interpolation method, in the previous context, is the space Z_2. Iterating the process, Carro et alt. obtained twisted sums of Z_2 with l_2. Looking at the complex diagram obtained, these are the faces of a cube formed by exact sequences. If, however, one looks inside the cube then twisted sums of Z_2 with itself appear. These are the first examples of twisted twisted Hilbert spaces that are not twisted Hilbert spaces (see F. Cabello [Twisted Hilbert spaces. Bull. Austral. Math. Soc. 59 (1999) 177–180] for a previous attempt and results in this direction).

Twisted sums of Banach spaces come defined by a certain type of nonlinear maps $F : X \curvearrowright X$ called z-linear. On Banach spaces with unconditional basis Kalton and Peck gave a natural method to construct such maps. The interpolative procedure is interesting because, besides its simplicity, provides an explicit computation of the associated z-linear map $F : Z_2 \curvearrowright Z_2$; and this is remarkable due to the absence of unconditionality in the basis of Z_2.

Q_p and $Q_p^\#$ for Riemann surfaces

by

Huaihui Chen

The Q_p spaces, $0 \le p \le \infty$, are conformally invariant Hilbert spaces of analytic functions which were introduced in the 1990's, first for functions in the unit disk and later for functions on hyperbolic Riemann surfaces R. $Q_0(R)$ is the usual Dirichlet space, $Q_1(R)$ the space of analytic functions of bounded mean oscillation and $Q_\infty(R)$ the Bloch-type spaces on R, which is a subspace of the classical Bloch space generally and coincides with it when R is the unit disk.

It has been known for a while that if $\infty \ge p \ge q \ge 0$ then $Q_q(R) \subset Q_p(R)$, which is called "nesting property". We introduced norm squares B_p for Q_p with the property that $B_p(f) = 1$ for every p if R is the unit disk and f the identity function. The main result reported in this talk is a stronger version of the nesting property: For every hyperbolic Riemann surfaces R and every analytic function f on R, we have $B_p(f) \le B_q(f)$ for any p, q with $\infty \ge p \ge q \ge 0$. Its proof makes essential use of the isoperimetric inequality.

$Q_p^\#$ are classes of meromorphic functions on a hyperbolic Riemann surface, which are defined by replacing the derivative of a analytic function by the spherical derivative of a meromorphic function. Similar but a littile bit complicate results have been obtained for $Q_p^\#$.

The results reported in this talk belong to a joint work with Rauno Aulaskari.

Composition operators on spaces of real analytic functions

by

Paweł Domański

This is a joint work with M. Langenbruch (Oldenburg).

Let $\varphi : M \to N$ be a real analytic map between real analytic manifolds. The corresponding composition operator $C_\varphi : A(N) \to A(M)$, $C_\varphi(f) := f \circ \varphi$, is continuous whenever the spaces of real analytic functions $A(M)$, $A(N)$ are equipped with their natural topologies.

We characterize when C_φ has closed image and we describe its image in that case under the additional assumption that φ is semi-proper, i.e., for every compact set $K \subseteq N$ there is a compact set $L \subseteq M$ such that $\varphi(L) = K$. The characterization is given in terms of the set $\varphi(M)$. One of the main tools is a very strong sequential density of polynomials in the space of real analytic functions over any compact set in $\mathbb{R}^d$ proved by the authors.

Moreover, we show that if $\varphi(M)$ is a coherent $\mathbb{C}$-analytic set then C_φ is open onto a closed subspace of $A(M)$.

Pseudodifferential operators on non quasianalytic classes of Beurling type

by

Carmen Fernandez Rosell

We introduce pseudodifferential operators (of infinite order) in the frame of non quasianalytic classes of Beurling type. We prove that such an operator with (distributional) kernel in a given Beurling class $\mathcal{D}'_{(\omega)}$ is pseudo-local and can be locally decomposed, modulo a smoothing operator, as the composition of a pseudodifferential operator of finite order and an ultradifferential operator with constant coefficients in the sense of Komatsu, both of them with kernel in the same class $\mathcal{D}'_{(\omega)}$. We also develop the corresponding symbolic calculus. Joint work with Antonio Galbis and David Jornet.

ω-hypoelliptic differential operators of constant strength

by

Antonio Galbis Verdú

We consider ω-hypoelliptic differential operators of constant strength. We show that any operator with constant strength and coefficients in $\mathcal{E}_\omega(\Omega)$ which is homogeneous ω-hypoelliptic is also σ-hypoelliptic for any weight function $\sigma = O(\omega)$. We also obtain some conditions on the weights (ω, σ)

to conclude that, for any operator $P(x, D)$ with constant strength, the σ-hypoellipticity of the frozen operator $P(x_0, D)$ implies the ω-hypoellipticity of $P(x, D)$. Joint work with Carmen Fernández and David Jornet.

Composition operators on uniform algebras, essential norms, and hyperbolically bounded sets

by

Pablo Galindo

The object of the talk is to relate the notion of "hyperbolic boundedness" to the essential spectrum of composition operators C_φ defined on a uniform algebra A. It is shown that the essential spectral radius of C_φ is strictly less than 1 if and only if the image of M_A under some iterate φ^n of φ is hyperbolically bounded. The set of composition operators is partitioned into "hyperbolic vicinities" that are clopen with respect to the essential operator norm.

This talk is based on joint work with Ted Gamelin and Mikael Lindström.

Commuting approximation properties

by

Gilles Godefroy

In several natural situations, approximation by finite rank operators can actually be achieved with a sequence of commuting operators. A striking example is provided by the Casazza-Kalton theorem stating that metric and commuting metric approximation properties are equivalent for every separable Banach space. It turns out that this equivalence extends to the unconditional metric approximation property. Proving this requires the use of weak* compactness in the bidual, and of several specific tools from the isometric theory of Banach spaces such as the "ball topology".

Boundary relations and Weyl families

by

Seppo Hassi

The concepts of boundary relations and the corresponding Weyl families are introduced. Let S be a closed symmetric linear operator (or, more generally, a closed symmetric relation) in a Hilbert space $\mathfrak{H}$, let $\mathcal{H}$ be an auxiliary Hilbert space, let $J_{\mathfrak{H}} = \begin{pmatrix} 0 & -iI_{\mathfrak{H}} \\ iI_{\mathfrak{H}} & 0 \end{pmatrix}$, and let $J_{\mathcal{H}}$ be defined analogously. A unitary relation Γ from the Kreĭn space $(\mathfrak{H}^2, J_{\mathfrak{H}})$ to the Kreĭn space $(\mathcal{H}^2, J_{\mathcal{H}})$ is called a *boundary relation* for the adjoint S^* if $\ker \Gamma = S$. The corresponding *Weyl family* $M(\lambda)$ is defined as the family of images of the defect subspaces $\widehat{\mathfrak{N}}_\lambda$ $(\lambda \in \mathbb{C}_+ \cup \mathbb{C}_-)$ under Γ. Here Γ need not be surjective and is allowed to be even multivalued. While this leads to fruitful connections between certain classes of holomorphic families of linear relations on the complex Hilbert space $\mathcal{H}$ and the class of unitary relations $\Gamma : (\mathfrak{H}^2, J_{\mathfrak{H}}) \to (\mathcal{H}^2, J_{\mathcal{H}})$, it also generalizes the notion of so-called boundary value space and extends essentially the applicability of abstract boundary mappings in the connection of boundary value problems. For instance, these new notions allow us to establish the following realization theorem: every $\mathcal{H}$-valued maximal dissipative (for $\lambda \in \mathbb{C}_+$) holomorphic family of linear relations is the Weyl family of a boundary relation, which is unique up to unitary equivalence if certain minimality conditions are satisfied. Further connections between analytic properties of Weyl families and geometric properties of boundary relations are investigated and some applications are shortly treated.

The talk is based on joint work with V.A. Derkach, M.M. Malamud, and H.S.V. de Snoo.

Extension of vector-valued functions

by

Enrique Jordá Mora

Results about the extension of functions defined on an open connected subset of R^n or C^n with values in a locally complete locally convex space E by means of weak extensions are presented. The functions for which the results are valid include several variable holomorphic functions and polyharmonic functions. The main tool is the representation of spaces of vector-valued functions as ε-products and Raikov's Theorem about the equivalence between stepwise closed and closed subspaces in (DFS) spaces. Work of Arendt and Nikolski and Grosse-Erdmann is extended, and an open question of Grosse-Erdmann is solved in the negative.

Joint work with J. Bonet and L. Frerick

Carleson measures and multipliers for diagonal Besov space

by

H. Turgay Kaptanoglu

We define and characterize Carleson and vanishing Carleson measures for diagonal Besov spaces B^p_q on the unit ball of $\mathbf{C}^N$. We do this by using the natural imbeddings from diagonal Besov spaces into Lebesgue classes in the form $(1-|z|^2)^t\, D^t f(z)$, where D^t is a radial differential operator of order $t \in \mathbf{R}$ and $q + pt > -1$. Our Carleson measures need not be finite, but our results extend those known for weighted Bergman spaces. In the course of our proofs, we find weakly convergent sequences in diagonal Besov spaces. As application, we characterize multipliers of diagonal Besov spaces.

Double singular integrals: interpolation and correction

by

S. V. Kislyakov

Let (Ω, μ) be a measure space, and let T be an operator bounded on $L^p(\mu)$ for all $p \in (1, \infty)$. Suppose that the expression Tf makes sense also for $f \in L^\infty$ and $f \in L^1$ (surely, Tf may leave these "extreme" spaces, e.g., it may be a distribution, etc.). Put $X_p = \{f \in L^p : Tf \in L^p\}$, $1 \leq p \leq \infty$.The question arises about the extent to which the scale X^p, including the points $p = 1, \infty$, inherits the poperties of the scale L^p.

Mainly, we are busy with interpolation properties and correction theorems. It is known that the scale X_p behaves nicely in these respects if T is a Calderón–Zygmund singular integral operator (CZO). It tuns out that, in some particular situations, the same can be said if T is a double CZO.

A prototypical example of a double CZO is the orthogonal projection of the space L^2 on the two-dimensional torus onto the Hardy space H^2 of functions analytic in two variables, and it has rather long been known that in this case the behaviour of the scale $X_p = H^p$ is also nice in the above sense. The purpose of the talk is to show that techniques used in the case of the torus are applicable to some other operators (which also can be treated as double singular integrals). For example, the Hardy–Littlewood square function in the case of dimension 1 fits into this pattern, as also do some multipliers (on the line or the circle) that fail to be CZO.

This is a joint work with D. S. Anisimov.

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Composition operators on vector-valued $BMOA$

by

Jussi Laitila

I discuss analytic composition operators C_φ on $BMOA(X)$, the space of X-valued analytic functions of bounded mean oscillation, where X is a complex Banach space.

The main result gives a sufficient condition for a composition operator to be weakly compact on $BMOA(X)$. As a consequence the weakly compact composition operators C_φ on $BMOA(X)$ are characterized under some restrictions on φ. The proof extends the characterization due to W. Smith (1999) of the compact composition operators on scalar-valued $BMOA$ and it is based on vector-valued methods due to Liu, Saksman and Tylli (1998).

A concrete example is given showing that $BMOA(X)$ differs from $wBMOA(X)$, a weak version of vector-valued $BMOA$, for infinite dimensional Banach spaces X. Composition operators on a class of weak vector-valued analytic function spaces, which includes $wBMOA(X)$, were considered by Bonet, Domański and Lindström (2001).

Inheritance of surjectivity for partial differential operators on spaces of real analytic functions

by

Michael Langenbruch

For an open set $\Omega \subset \mathbb{R}^n$ let $A(\Omega)$ be the space of real analytic functions on Ω. We will present a new quantitative characterization of the partial differential operators $P(D)$ with constant coefficients which are surjective on $A(\Omega)$. This allows to study the inheritance of the surjectivity of $P(D)$ on $A(\Omega)$ more closely. Especially, we prove that $P(D)$ is surjective on $A(\mathbb{R}^n)$ if $P(D)$ is surjective on $A(\Omega)$ for some $\Omega \neq \emptyset$. For convex Ω, this was shown by Hörmander with a proof which cannot be transferred to the general situation.

Approximation properties and ideals of operators

by

Vegard Lima

We give characterizations of Grothendieck's metric approximation property (MAP) and approximation property (AP) in terms of ideals of operators.

A subspace E of a Banach space F is an *ideal* in F if there is a *Hahn-Banach extension operator* $\phi : E^* \to F^*$, that is if ϕ satisfies $(\phi e^*)(e) = e^*(e)$ and $\|\phi e^*\| = \|e^*\|$ for all $e \in E$ and $e^* \in E^*$.

A Banach space X has the MAP if and only if $\mathcal{F}(Y, X)$, the space of all finite rank operators, is an ideal in $\mathcal{L}(Y, X)$, the space of all bounded operators, for every Banach space Y. A dual space X^* has the MAP if and only if $\mathcal{F}(X, Y)$ is an ideal in $\mathcal{L}(X, Y)$ for every Banach space Y.

Pushing these ideas further we show that X has the MAP if and only if $\mathcal{F}(Y, X)$ is an ideal in $\mathcal{L}(Y, X^{**})$ for all Banach spaces Y. Furthermore, X^* has the MAP if and only if for all Banach spaces Y and all Hahn-Banach extension operators $\phi : X^* \to X^{***}$ there exists a Hahn-Banach extension operator $\Phi : \mathcal{F}(Y, X)^* \to \mathcal{L}(Y, X^{**})^*$ such that $\Phi(x^* \otimes y^{**}) = (\phi x^*) \otimes y^{**}$ for all $x^* \in X^*$ and all $y^{**} \in Y^{**}$.

Similarly we show that X^* has the AP if and only if for all Banach spaces Y and all Hahn-Banach extension operators $\phi : X^* \to X^{***}$ there exists a Hahn-Banach extension operator $\Phi : \mathcal{F}(Y, X)^* \to \mathcal{W}(Y, X^{**})^*$ such that

$\Phi(x^* \otimes y^{**}) = (\phi x^*) \otimes y^{**}$ for all $x^* \in X^*$ and all $y^{**} \in Y^{**}$, which in turn is equivalent to $\mathcal{F}(Y, \hat{X})$ being an ideal in $\mathcal{W}(Y, \hat{X}^{**})$ for all Banach spaces Y and all equivalent renormings $\hat{X}$ of X.

Representation of orthogonally-additive polynomials on Banach lattices

by

Jose Llavona

The main result is a representation theorem for n-homogeneous orthogonally-additive polynomials on Banach lattices. This theorem is used to study the linear span of the set of zeros of homogeneous real-valued orthogonally-additive polynomials. We also indicate how these results can be used to study weak topologies induced by orthogonally-additive polynomials on Banach lattices. All these results are included in a joint paper with Yoav Benyamini and Silvia Lassalle.

On the isomorphism classes of weighted spaces of harmonic and holomorphic functions

by

Wolfgang Lusky

Let Ω be either the complex plane or the open unit disc. Moreover, let v be a radial weight function on Ω. Then we completely determine the isomorphism classes of

$$Hv = \{\ f : \Omega \to \mathbb{C} \ \text{ holomorphic } \ : \ \sup_{z\in\Omega} |f(z)|v(z) < \infty\ \}$$

and investigate some isomorphism classes of

$$hv = \{\ f : \Omega \to \mathbb{C} \ \text{ harmonic } \ : \ \sup_{z\in\Omega} |f(z)|v(z) < \infty\ \}.$$

Our main results show that, without any further condition on v, there are only two possibilities for Hv, namely either $Hv \sim l_\infty$ or $Hv \sim H_\infty$, and at least two possibilities for hv, again $hv \sim l_\infty$ and $hv \sim H_\infty$. We also discuss many new examples of weights including an example where both, Hv as well as hv, are isomorphic to H_∞.

On holomorphic functions attaining their norms

by

Manuel Maestre

We study sufficient conditions for the functions uniformly continuous on the closed unit ball and holomorphic on the open unit ball of a complex Banach space X that attain their norms are dense. provided that X has the Radon-Nikodym property. For $X = d_*(\omega, 1)$, a predual of a Lorentz sequence space, we prove that the product of two polynomials with degree less than or equal two attains its norm if, and only if, each polynomial attains its norm.

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Joint work with María D. Acosta, Jerónimo Alaminos and Domingo García.
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Composition operators on the Bloch space

by

Shamil Makhmutov

Let $\mathcal{B}$ be the Bloch space of analytic function in the unit disk D and X be one the following spaces: $\mathcal{B}$, or analytic Besov space B_p, or Q_p space, or H_p. We show that composition operators $C_\varphi : \mathcal{B} \to X$ are induces by mappings φ from corresponding hyperbolic classes X^h of analytic self-mappings of the unit disk. We study properties of function φ from the classes X^h, conditions of compactness of C_φ and estimate the essential norm of C_φ.

Lipschitz quotient mappings between finite-dimensional spaces

by

Olga Maleva

Abstract: A mapping between Banach spaces is called a Lipschitz quotient mapping, if the image of any ball of arbitrary radius r and arbitrary center x is contained in a ball of radius Lr about the image of x and contains a ball of radius cr about the image of x, for some universal positive constants L and c. A natural question arises: how complicated can such mappings be? In this talk, I will present a number of results which describe Lipschitz quotient mappings and their generalisations between finite-dimensional spaces.

Geometric characterizations of the Daugavet property in C*-algebras and von Neumann preduals

by

Miguel Martin
(joint work with Julio Becerra)

A Banach space is said to have the Daugavet property if

$$\|Id + T\| = 1 + \|T\| (DE)$$

for every rank-one operator $T \in L(X)$. In such a case, all weakly compact operators on X also satisfy (DE).

We give geometric characterizations of this property in the settings of C^*-algebras and von Neumann preduals as follows:

-For a C^*-algebra A, the following are equivalent:

(i) A has the Daugavet property.

(ii) The norm of A is extremely rough.

(iii) The norm of A is not Frchet-smooth at any point.

-Let A be the predual of a von Neumman algebra. Then, the following are equivalent:

(i) A has the Daugavet property.

(ii) Every weak-open subset of the unit ball of A has diameter 2.

(iii) The unit ball of A is not dentable.

(iv) The unit ball of A has no extreme points.

We also show that, for C^*-algebras and von Neumann preduals, the Daugavet property is equivalent to an stronger property called the Uniform Daugavet property.

Chaos for power series of backward shift operators

by

Felix Martinez Gimenez

We study the chaotic behavior of formal power series of backward shift operators on Köthe echelon sequence spaces. This chaotic behavior has a close relationship with the point spectrum of the operator and the range of the power series.

Given a Köthe matrix $A = (a_{i,k})_{i,k\in\mathbb{N}}$, we consider the echelon sequence space

$$\lambda_p(A) := \left\{ x \in \mathbb{C}^{\mathbb{N}} \ : \ \|x\|_k := \left(\sum_{i=1}^{\infty} |x_i \ a_{i,k}|^p \right)^{1/p} < \infty, \ \forall \ k \in \mathbb{N} \right\} .$$

For a sequence $w_i \neq 0, i = 2, 3, \ldots,$ of complex numbers (called weight sequence), its associated weighted backward shift B_w is defined as

$$B_w(x_1, x_2, x_3, \ldots) := (w_2x_2, w_3x_3, w_4x_4, \ldots) .$$

Given a formal power series $f(z) = \sum_{j=0}^{\infty} f_j z^j$, we consider the operator $f(B_w) := \sum_{j=0}^{\infty} f_j B_w^j$, provided $f(B_w) : \lambda_p(A) \to \lambda_p(A)$ is well defined and continuos. The study of hypercyclicity and chaos for operators $f(B_w)$ was initiated for the particular cases of weighted backward shift B_w and perturbations of the identity by weighted backward shift $I + B_w$. Complete (and computable) characterizations were obtained for these cases. Conditions under which $f(B_w)$ is chaotic will be shown. Applications to natural spaces such as the space of entire functions on the complex plane and Hilbert spaces of entire functions are included.

We also present an open question concerning the existence of chaotic operators on nuclear Fréchet spaces (with basis).

Part of this talk is based on joint work with A. Peris.

Optimal Gevrey classes for the existence of solution operators for linear partial differential operators

by

Reinhold Meise

We report on joint work with R. W. Braun (Düsseldorf) and B. A. Taylor (Ann Arbor).

The main aim of the lecture is to give the idea of proof of the following theorem.

Theorem. Assume that for $P \in \mathbb{C}[z_1, z_2, z_3] \setminus \mathbb{C}$ the differential operator $P(D)$ admits a solution operator on $\mathcal{D}'_{(\omega)}(\mathbb{R}^3)$ for some weight function ω. Then there exists a rational number $b \in \ [0, 1[$ such that $P(D)$ admits a solution operator on $\mathcal{D}'_{(\sigma)}(\mathbb{R}^3)$ if and only if $t^b = O(\sigma(t))$ as t tends to infinity.

Besides this it will be shown how this number b can be computed in an "algorithmic" way.

The Banach space $D(0, 1)$ is primary

by

Artur Michalak

$D(0, 1)$ is the Banach space (under the sup norm) of all scalar functions defined on the interval $[0, 1)$ that are right-continuous at each point of $[0, 1)$, and have a left-hand limit at each point of $(0, 1]$. The space has appeared in the theory of Banach spaces as an example of a Banach space with some interesting properties. Thus, for example, H. Corson [1] showed that the quotient space $D(0,1)/C([0,1])$ is isomorphic to $c_0([0,1])$ and also that the space $D[0,1)$ is not normal in its weak topology. Every isometric copy of c_0 in $D(0,1)$ is complemented (see [4]). It is well known that, isometrically, the space $D(0,1)$ can be identified with the space $C(K)$, where K is the two arrows space. The space plays a crucial rule in the description of Banach spaces X for which there exists an increasing function from $[0,1]$ into X with uncountable set of points of discontinuity (see [2]).

The lecture is devoted to the proof of the primariness of $D(0,1)$. A partial result in this direction was shown by W. Patterson in [4]. She showed that whenever $D(0,1)$ has Banach direct sum decomposition $D(0,1) = X \oplus Y$ where Y is isometrically isomorphic to c_0, then X is isomorphic to $D(0,1)$. The proof of primariness of $D(0,1)$ is divided into a few steps. Two of them seems to be the crucial. The first is to show that for every continuous linear operator $J : D(0,1) \to D(0,1)$ with nonseparable range there exists a subspace Y of $D(0,1)$ which is isomorphic to $D(0,1)$, the restriction $J|_Y$ is an isomorphism, and there exists a closed and uncountable subset S of the two arrows space and continuous map $\eta : S \to (D(0,1)^*, w*)$ such that $\sup\{|\eta(l)(J(y))| : l \in S\} = \|y\|$ for every $y \in Y$. The second is to show that if $J : C(S) \to D(0,1)$ is an isometric embedding where S is a closed and uncountable subset of the two arrows space, then there exists a closed subspace Z of $J(C(S))$ which is isomorphic to $D(0,1)$ and complemented in $D(0,1)$. This part of our considerations is closely related to a result of A. Pełczyński [5, Thm. 1a].

References

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Factorization of weakly compact sets of Banach spaces of operators

by

Kristel Mikkor

We prove uniform factorization results that describe the factorization of compact sets and weakly compact operators via Hölder continuous homeomorphisms having Lipschitz continuous inverses. This yields, in particular, quantitative strengthenings of results of Graves and Ruess on factorization through ℓ_p-spaces and of Aron, Lindström, Ruess and Ryan on the factorization through universal space of Figiel and Johnson. Our method is based on the isometric version of the Davis-Figiel-Johnson-Pełczyński factorization construction due to Lima, Nygaard and Oja. This is a joint work with Eve Oja.

Vector-valued versions of Sobczyk's theorem

by

Yolanda Moreno Salguero

The main goal of this talk is to present vector-valued versions of Sobczyk's theorem. Recall that Sobczyk's theorem can be read as follows: every w^*-null sequence of functionals $(y_n^*)_n$ on a Banach space Y can be extended to any space X such that X/Y is separable by a w^*-null sequence of functionals. We introduce a device which provides two types of vector valued versions of this result.

a) Asking the quotient space to have the Bounded Approximation Property (BAP) we get: *Every pointwise null sequence of operators $T_n : Y \to Y_n$ admitting uniformly bounded extensions to a space X such that X/Y is separable with the BAP can be extended to X by a pointwise null sequence of operators.*

b) Asking the range spaces (Y_n) to be sparably injective we get: *If* (Y_n) *is a sequence of uniformly separably injective spaces then the vector sum* $c_0(Y_n)$ *is separably injective.* This last formulation provides new proofs for Sobczyk's theorem but also for the generalizations of Rosenthal and Oikhberg-Johnson.

Further applications to the classical problem of extension of $C(K)$-valued operators are presented.

Regularity of measures, entropy and the law of the iterated logarithm

by

Artur Nicolau

We study regularity properties of a positive measure in the euclidean space, such as being absolutely continuous or singular with respect to certain Hausdorff measures, in terms of its doubling behaviour. Joint work with Jose Gonzalez Llorente.

Boundary correspondence of the Nevanlinna counting function

by

Pekka J. Nieminen

Let ϕ be an analytic self-map of the unit disc $\mathbf{D}$ and let C_ϕ be the induced composition operator on the Hardy space H^2. Two rather different expressions have been found for the essential norm of such an operator. The first, due to J. H. Shapiro (1987), involves the Nevanlinna counting function N_ϕ associated to ϕ, and the second, due to J. A. Cima and A. L. Matheson (1997), utilizes the singular components σ_α ($\alpha \in \partial\mathbf{D}$) of the Aleksandrov measures of ϕ. Combining these expressions yields the puzzling function-theoretic equality

$$\limsup_{|w|\to 1-} \frac{N_\phi(w)}{1-|w|} = \sup_{|\alpha|=1} \|\sigma_\alpha\|.$$

Our main result is a local version of this equality. We introduce a measure-valued (and normalized) refinement of the Nevanlinna counting function and establish that it converges weak-star to σ_α as α is approached non-tangentially off a small exceptional set.

This is joint work with Eero Saksman (University of Jyväskylä).

Stationary and nonstationary biorthogonal compactly supported wavelets

by

I.Ya. Novikov

Basis and approximation properties of stationary and nonstationary biorthogonal compactly supported wavelets in different function spaces are considered. In particular stationary biorthogonal compactly supported wavelets preserving localization with the growth of smoothness is examined. Two examples of infinitely differentiable nonstationary biorthogonal compactly supported wavelets are investigated also.

On interpolating and sampling sequences in Bergman spaces

by

Maria Nowak
(Artur Kukuryka, Maria Nowak and Pawel Sobolewski)

Let A^p, $0 < p < \infty$, denote the classical Bergman space in the unit disk $\mathbb{D}$. A sequence $\{z_k\}$ of distinct points in the unit disk is called an interpolation sequence for A^p, if the interpolation problem $f(z_k) = w_k$ for $k = 1, 2...$ has a solution $f \in A^p$ provided $\sum_{k=1}^{\infty}(1-|z_k|^2)^2|f(z_k)|^p < \infty$. A sequence $\{z_k\}$ of distinct points in the unit disk is called a sampling sequence for A^p if there exist positive constants K_1, K_2 such that

$$K_1\|f\|_p^p \le \sum_{k=1}^{\infty}(1-|z_k|^2)^2|f(z_k)|^p \le K_2\|f\|_p^p.$$

Sufficient and necessary conditions for a sequence to be interpolation or sampling for A^p are given in terms of pseudohyperbolic densities. These results are due to Seip for the case $p = 2$. Schuster and Varolin have extended these results to general values of p.

Let ρ denote the pseudohyperbolic metric given by $\rho(z, \zeta) = \left|\frac{\zeta - z}{1-\bar{\zeta}z}\right|$. We say that a sequence of points $\Gamma = \{z_n\}$ in $\mathbb{D}$ is uniformly discrete if $\delta(\Gamma) = \inf_{j\neq k} \rho(z_j, z_k) > 0$. One of the important results connected with the characterization of interpolation and sampling sets is the following. Assume that a uniformly discrete sequence Γ admits an analytic function g with the property

$$|g(z)| \simeq \rho(z, \Gamma)(1 - |z|^2)^{-\alpha}, \quad z \in \mathbb{D} \tag{1}$$

for some $\alpha > 0$. Then a sequence Γ is an interpolation sequence for A^p if and only if $\alpha < \frac{1}{p}$, and Γ is a sampling sequence for A^p if and only if $\alpha > \frac{1}{p}$.

For $\mu > 1$ define Γ as the set of points equally spaced on the circles $|z| = \left(\frac{1}{\mu}\right)^{2^{-n}}$, $n = 1, 2, \ldots$, such that $z^{2^n} = \frac{1}{\mu}$. We show directly that the function

$$g(z) = \prod_{n=1}^{\infty} \frac{z^{2^n}\mu - 1}{1 - \frac{1}{\mu}z^{2^n}},$$

considered by Horowitz in his paper (J.Analyse Math. 62 (1994)), satisfies condition (1) with $\alpha = \frac{\log \mu}{\log 2}$.

Elastic Banach spaces and the distance between isomorphic separable spaces

by

E. Odell
(joint with W.B. Johnson)

A Banach space X is elastic if for some $K < \infty$, whenever Y embeds into X there exists $Z \subseteq X$ with $d(Y, Z) \leq K$. Thus there exists an isomorphism $T : Y \to Z$ with $\|T\| \, \|T^{-1}\| \leq K$. We prove that a separable infinite dimensional elastic Banach space must contain an isomorph of c_0. This proof uses an ordinal index of J. Bourgain. We then use this result to prove that if X is separable and infinite dimensional, then for all $M < \infty$ there exists a Banach space Y, isomorphic to X, with $d(Y, X) > M$.

Topological structure of the space of weighted composition operators on H^∞

by

Shuichi Ohno

We study properties of the topological structure of the space of weighted composition operators on the space of bounded analytic functions on the open unit disk in the uniform operator topology. Moreover, we characterize the compactness of differences of two weighted composition operators. This is a joint-work with T. Hosokawa and K. Izuchi.

Metric approximation properties and trace mappings

by

Eve Oja

We establish necessary and sufficient conditions involving trace mappings and Hahn-Banach extension operators for a Banach space to have metric or metric compact approximation properties. We also study metric approximation properties for dual spaces. As an application, alternative (hopefully enlightening) proofs are given for the well-known result that the dual space has the metric approximation property whenever it has the approximation property and the Radon-Nikodým property. This is joint work with Åsvald Lima.

Lacunary statistical convergence in finite dimensional spaces

by

Serpil Pehlivan

Abstract: In this note we introduce the concept of lacunary statistical cluster (l.s.c.) point and to prove some of its properties in finite dimensional Banach spaces. We also show that the set N_x^θ of l.s.c. points is nonempty and compact.

Hypercyclic and chaotic semigroups of operators

by

Alfredo Peris

This talk is based on joint work with J. A. Conejero.

We consider strongly continuous semigroups of operators $\{T_t : X \to X\}_{t\in\Delta}$ defined on separable $\mathcal{F}$-spaces X, where the index set Δ is either $\mathbb{R}$, $\mathbb{R}^+$, or a sector in the complex plane.

We study the properties of hypercyclicity and chaos for semigroups of operators. The semigroup $\{T_t\}_{t\in\Delta}$ is called *hypercyclic* if there exist $x \in X$ so that the orbit $\{T_t x \ / \ t \in \Delta\}$ is dense in X. $\{T_t\}_{t\in\Delta}$ is *chaotic* if it is hypercyclic and, in addition, the set of periodic points $\mathcal{P} := \{x \ \in X \ / \ \exists t \in \Delta, \ t \neq 0 : \ T_t x = x\}$ is dense in X. A single operator $T : X \to X$ is hypercyclic if the are $x \in X$ whose orbit $\{T^n x \ / \ n \in \mathbb{N}\}$ is dense in X. Hypercyclic operators T which admit a dense set of periodic vectors $\mathcal{P} := \{x \in X \ / \ \exists n \in \mathbb{N} : \ T^n x = x\}$, are called chaotic.

We analyze, for hypercyclic semigroups $\{T_t\}_{t\in\Delta}$, the hypercyclicity of the single operators T_t. Some different phenomena are established depending on the index set Δ. Special type of semigroups, namely translation semigroups on weighted spaces of p-integrable functions defined on sectors of the complex plane, are studied. We obtain characterizations of hypercyclicity and chaos for this type of semigroups and provide several examples exhibiting different behaviour.

Massiveness of sets in Banach spaces and weak integrability of vector-valued functions

by

Märt Põldvere

We characterize the concepts of thick and weak*-thick subsets of Banach spaces (introduced by V.P. Fonf and M.I. Kadets) in terms of weak integrability of vector-valued functions and of boundedness of vector measures. This is joint work with T.A. Abrahamsen and O. Nygaard.

Differentiability of Lipschitz mappings in Banach spaces

by

David Preiss

The first part of the talk will review the main points of what is known about existence of points of Frechet differentiability of Lipschitz functions on Banach spaces. The second part will try to describe possible ways of attacking the problem of existence of common points of Frechet differentiability of several Lipschitz functions on a Hilbert space.

On inverse scattering from random potentials

by

Eero Saksman

The talk describes a joint work with Matti Lassas and Lassi Päivärinta, University of Helsinki. Let q be a compactly supported (random) potential on the plane $\mathbf{R}^2$. Assume that $x_0 \in \mathbf{R}^2$ lies outside the support of q. We consider inverse Schrödinger back-scattering from the potential q with incident fields corresponding to point sources nearby x_0.

The class of potentials q considered includes locally isotropic non-smooth Markov fields. By letting C_q stand for the covariance operator of q, the principal part of the inverse operator $(C_q)^{-1}$ has the form $a\Delta$. The function a can be thought to describe the material parameters.

Our main result implies that almost surely one can recover the coefficient a from energy averages of the measurements over different frequences, using only back-scattering data near the point x_0. Somewhat surprisingly, this can be be done by using measurements from a single realization of the potential q only. Our analysis treats the full non-linear problem without employing approximative assumptions.

On the dimension of right ideals in the second dual of the group algebra

by

Pekka Salmi

This talk deals with the second dual $L^1(G)^{**}$ (equipped with the first Arens product) of the group algebra of a locally compact group G. The first Arens product on $L^1(G)^{**}$ can be seen as the unique extension of the convolution on the weak*-dense subspace $L^1(G)$ of $L^1(G)^{**}$ such that all the right translates are weak*-continuous. However, the only left translates that are weak*-continuous are the ones determined by members of $L^1(G)$ – a fact that makes the study of $L^1(G)^{**}$ challenging.

We consider the ideal structure of the Banach algebra $L^1(G)^{**}$ and show that every nontrivial right ideal has dimension at least 2^{2^κ} (for noncompact G), where κ is the smallest cardinality of the covering of G by compact sets. To prove this result we apply a factorization theorem of L^∞ functions. This method works even in the more general case when the group algebra is equipped with a diagonally bounded weight function. This is a joint work with Mahmoud Filali and John Pym.

Convergence to the Denjoy-Wolff point

by

Joel Shapiro

This is joint work with Paul Bourdon and Valentin Matache. We examine the way in which a holomorphic self-map of the unit disc converges to its Denjoy-Wolff point, with emphasis on convergence a.e. and in the H^2 norm. For inner functions there are interesting connections with ergodic theory.

Nonlinear parabolic equations in Colombeau algebras

by

Mirjana Stojanovic

Abstract: We give the existence-uniqueness theorems in Colombeau algebras of generalized functions for nonlinear parabolic equations with strongly singular initial data and nonlinear free term. We use the regularization with delta sequences. We give the regularization for heat semigroup with respect to the time variable t or we do the regularization of nonlinear term with respect to the space variable. Following the different growth of delta sequences we find the link between the singularity of the nonlinear term, initial data and semigroup. We transfer these results for to nonlinear Schrödinger equation.

Fredholm theory of Toeplitz operators on the Hardy space H^1

by

Jani Virtanen

The Fredholm properties of Toeplitz operators T_a on Hardy spaces H^p $(1 < p < \infty)$ with continuous symbols a are well understood. We consider T_a acting on H^1, where the operator is bounded provided that a belongs to the class of symbols given by Janson and Stegenga's result on the pointwise multipliers in H^1. A necessary and sufficient condition for Fredholmness of T_a is given when a is continuous and satisfies a mild additional condition (much weaker than Hölder continuity). A formula for the index of T_a is also derived. In addition, we study the case of matrix-valued symbols and Toeplitz operators on BMO_A.

Splitting of differential complexes

by

Dietmar Vogt

We discuss the splitting of a short exact sequence

$$0 \longrightarrow G \longrightarrow E \xrightarrow{S} F \longrightarrow 0$$

of nuclear Fréchet spaces, where F need not admit a continuous norm, i.e. E or F may be spaces of smooth functions or sections of smooth bundles on an open subset of $\mathbb{R}^n$ or on a σ-compact C^∞-manifold. If E is such a space then we give a necessary and sufficient condition for the splitting of the sequence, in terms of a condition on S and of linear topological invariants (Ω_{loc}) and $(\mathrm{DN}_{\mathrm{loc}})$. This is used to study the splitting of differential complexes and to give a condition for splitting in terms of $P(D)$-convexity with bounds, so generalizing results of R. Meise, B. A. Taylor, D. Vogt and V. P. Palamodov. It can also be used to give a structure theory of the space $s^{\mathbb{N}}$, a space which is isomorphic to many spaces of smooth functions. We present a different approach to problems which have been studied in P. Domański, D. Vogt, *A splitting theorem for the space of smooth functions*, J. Funct. Anal. **153** (1998), 203-248.

A Banach space with the Schur and the Daugavet property

by

Dirk Werner

A Banach space X is said to have the *Daugavet property* if every rank-1 operator $T: X \to X$ satisfies

$$\|\mathrm{Id} + T\| = 1 + \|T\|.$$

We show that a minor refinement of the Bourgain-Rosenthal construction of a Banach space without the Radon-Nikodým property which contains no bounded δ-trees yields a space with the Daugavet property and the Schur property. Using this example we answer some open questions on the structure of such spaces; for example, the Daugavet property is not inherited by ultraproducts.

Greedy bases in r.i. spaces

by

Przemyslaw Wojtaszczyk

It is known that the Haar system is a greedy basis in L_p for $1 < p < \infty$. We show that L_p spaces are the only r.i. spaces where Haar system is greedy. We show that greedy basis in Lorentz space $L_{p,q}$ has to be very strange, if it exists at all. On the other hand there exists examples (constructed by Johnson, Maurey, Schechtman and Tzafriri) of r.i. spaces with symmetric so greedy bases.

Some results on Q_K spaces

by

Hasi Wulan

Starting from a nondecreasing function $K : [0, \infty) \to [0, \infty)$, we introduce a Möbius-invariant Banach space Q_K of functions analytic in the unit disk in the plane. We give a general theory of these spaces which gives most basic properties of Q_p-spaces. We obtain necessary and sufficient conditions on K to have $Q_K = \mathcal{B}$ or $Q_K = \mathcal{D}$, where the Bloch space $\mathcal{B}$ and the Dirichlet space $\mathcal{D}$ is the largest and the smallest space of Q_K-type, respectively. We also give a general way to construct a space Q_{K_0} which is between the BMOA space and the Bloch space; that is $BMOA \subsetneqq Q_{K_0} \subsetneqq \mathcal{B}$. We give a mild integral condition on K, which is sufficient and the best possible to ensure that f is a Bloch function if and only if f belongs to Q_K. For example, for areally mean q-valent functions. As corollaries, we give an extended version of Pommerenke's theorem and the proof presented is independently developed. The corresponding results for meromorphic case are given.