Optimized entropy-constrained vector quantization of lossy vector map compression

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Abstract

Quantization plays an important part in lossy vector map compression, for which the existing solutions are based on either a fixed size open-loop codebook, or a simple uniform quantization. In this paper, we proposed an entropy-constrained vector quantization to optimize both the structure and size of the codebook at the same time using a closed-loop approach. In order to lower the distortion to a desirable level, we exploit two-level design strategy, where the vector quantization codebook is designed only for most common vectors and the remaining (outlier) vectors are coded by uniform quantization.

1. Introduction

Vector maps consist of geographic information such as waypoints, routes and areas, which can be represented as a sequence of points in a given coordinate system. To reduce the archive space and transmission time, a variety of compression algorithms have been investigated and developed for compressing vector maps [2-6]. Existing lossy compression algorithms use two different strategies: *polygonal approximation* and *quantization-based method*.

In *polygonal approximation,* the number of points is reduced and the curve represented by a coarser approximation [1]. In *quantization-based method*, differential coordinates of subsequent sampling points are considered as the prediction error and these residual vectors are quantized using various methods, such as *uniform quantization* [2], *product scalar quantization* [3] *and vector quantization* with fixed size codebook [4]. In [5], *reference line* method first identified a series of references lines by polygonal approximation and then estimated prediction errors for the remaining points according to their nearest reference lines followed by *product scalar quantization* in a similar manner to [3]. In [6], *dynamic quantization* was studied where the curve approximation was performed by taking into consideration vector quantization of the approximation line segments.

In this paper, we propose three concrete improvements for the *quantization-based method*. Firstly, all the previous methods use a fixed size codebook, whereas *entropy-constrained pair-wise nearest neighbor for vector quantization* (ECPNN-VQ) is used that optimizes the size of the codebook as well. As a merge-based clustering method, ECPNN-VQ will stop reducing the size of the codebook when a given bit-rate constraint is met.

Secondly, the codebook is further optimized where the codebook of vector quantization is only applied for most common vectors and the rest (*outliers*) are processed by uniform quantization. In contrary to typical image compression, vector data has a wide dynamic range that can vary significantly from a dataset to another. In conventional approaches, a large-size codebook has therefore been required in order to achieve lower distortion. In practice, however, the code length and cost of the codebook itself must also be taken into account, which is ignored in the existing vector quantization methods. In specific, large-size codebooks are required when high bit rate is desired, and it is difficult to achieve a desirable compression performance due to the additional code length of the codebook. To attack this problem, an additional *"outliers"* cluster is designed for vectors that differ too much from the majority of the vectors. Those vectors with significantly high cost in the rate-distortion sense are selected as *"outliers"* and coded by a separate escape codeword. The underlying quantization method leads to a robust compression performance both for high and low bit-rates.

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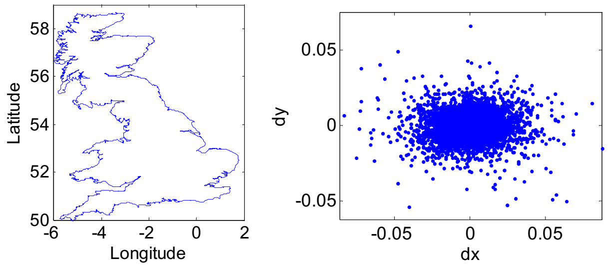

Fig. 1, Test map Britain with 10,910 points (left), and the prediction residuals with MSE $=1.8 \cdot 10^{-4}$.

Finally, the close-loop structure in the encoding step was improved by dynamic programming algorithm. The remainder of the paper is organized as follows. The proposed method is introduced in section 2; Section 3 reports the corresponding experimental results; and conclusions are drawn in section 4.

2. Proposed Method

In vector map compression, we want to compress a given 2-D vector sequence: $P = (p_1, p_2, \ldots, p_n)$, under given bit-rate constraint c. General compression procedure contains three steps: *prediction*, *quantization* of the residual vectors and *entropy coding*. Differential coordinates of subsequent sampling points are used as the prediction error in prediction step. A sample test curve and the distribution of the corresponding differential coordinates for which the quantization codebook is designed, are shown in Fig. 1.

2.1 Initial the codebook in vector quantization

In vector quantization, we want to quantize the residual vectors by minimizing mean square error under a constraint that the average bit-rate does not exceed c:

$$\min D, s.t. R < c, where\ D = \sum_{i=1}^{N} (\| \boldsymbol{v}_i - Q(\boldsymbol{v}_i) \|^2) \tag{1}$$

$\boldsymbol{v}_i$ is the residual vector and $Q(\boldsymbol{v}_i)$ is its quantized form.

In entropy-constrained vector quantization (ECVQ), the problem can be solved by a *Lagrangian minimization procedure* by converting it as an unconstrained optimization problem [7], formulated as $J = D + \lambda R$. For each Lagrangian parameter λ, it has a corresponding point on the rate-distortion curve. However, unlike image coding, prediction error for vector data varies for different case, and thus, using a fixed size (k) in codebook design does not solve the problem efficiently. In order to find a better combination of λ and k, ECVQ can be applied but at the cost of higher time complexity, which makes it suitable only off-line.

Entropy-constrained pair-wise nearest neighbor for vector quantization (ECPNN-VQ) has been proposed in [8]. It merges the pair of cluster that results in the smallest increase in distortion and largest decrease in rate. The increased distortion after merging two clusters i and j can be calculated by:

$$\Delta D = \frac{n_i + n_j}{n_i n_j} \| \boldsymbol{c}_i - \boldsymbol{c}_j \|_2^2 \tag{2}$$

n_i and n_j are the number of vectors in cluster i and j, respectively, and $\boldsymbol{c}_i$ and $\boldsymbol{c}_j$ are their centroid vectors. The change in bit-rate can be calculated as:

$$\begin{aligned} \Delta R &= -n_i \log(n_i / n) - n_j \log(n_j / n) \\ &-(-(n_i + n_j) \log((n_i + n_j)/n) + r_q) \end{aligned} \tag{3}$$

r_q is the code length of one quantized vector in codebook, n is the number of residual vector.

In every merge step, the pair of clusters with minimum $-\Delta D/\Delta R$ is merged. This can also be considered as searching the minimum slope in the rate-distortion curve, and thus, it guarantees the optimality of each merge step. Since in classic ECVQ framework, λ is interpreted as the slope of the line supporting the operational rate-distortion curve, and therefore, in ECPNN-VQ, it is approximated by:

$$\lambda \approx -(D^{n+1} - D^n)/(R^{n+1} - R^n). \tag{4}$$

The time complexity of ECPNN-VQ is $O(\tau N^2)$, the same as that of the traditional PNN algorithm [9].

2.2 Optimize quantization by outlier cluster

After ECPNN-VQ, the cost of each residual vector i in cluster j can be formulated as:

$$J_{ij} = \| \boldsymbol{v}_i - \boldsymbol{c}_j \|_2^2 + \lambda(-\log_2(n_j / n) + r_q / n_j) \tag{5}$$

If a very high accuracy is needed, a large-sized codebook is intractable in achieving a desirable coding efficiency. Better compression performance can be achieved by uniform quantization because no overhead is needed for storing the codebook. Therefore, we apply two-level codebook so that the most common vectors are coded by vector quantization using the optimized codebook, while the outlier vectors are coded by uniform quantization. In uniform quantization, given quantization level l, residual vector $\boldsymbol{v}_i$ is quantized as $Q(\boldsymbol{v}_i) = ([\Delta x_i/l] \cdot l, [\Delta y_i/l] \cdot l)$, where $q_{xi} = [\Delta x_i/l]$, $q_{yi} = [\Delta y_i/l]$ are integer value be coded.

The mean square error D_0 can be calculated by:

$$\int_0^{l/2} \frac{1}{l} x^2 dx + \int_{l/2}^{l} \frac{1}{l} (l - x)^2 = \frac{1}{2} D_0 \tag{6}$$

We have $D_0 = l^2/6$.

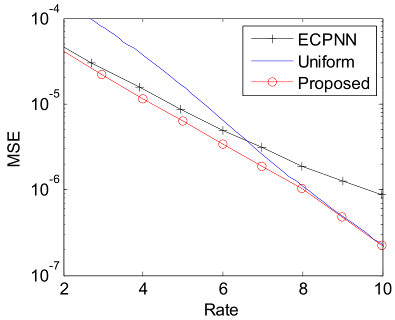

Fig.2, Comparison of the vector quantization of the residual vectors

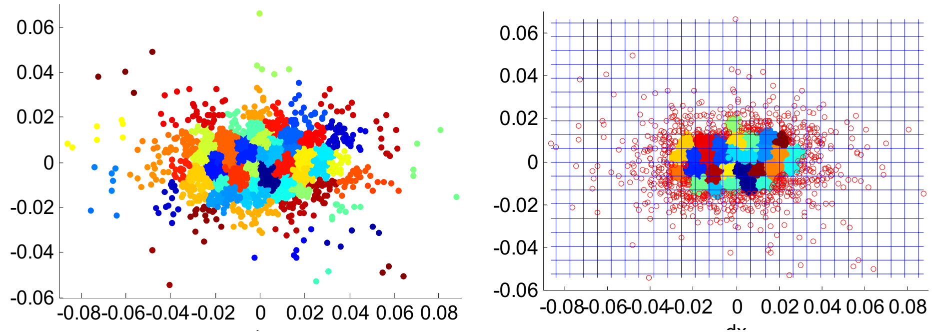

Fig. 3, Demonstration of the outlier selection (5 bits/point constraint). ECPNN with MSE= 8.7·10^{-6} and codebook size 78(left). The proposed two-level codebook with MSE= 6.9·10^{-6} and size 30 (right). Outliers are marked as 'o'. Grid size for uniform quantization is also labeled.

We observe that the $|q_x|$ and $|q_y|$ can be described by *geometric distribution*:

$$f(|q_x|) = (1-p_x)^{|q_x|} p_x \tag{7}$$

where p_x is the parameter, which is approximated by:

$$p_x = 1/(\frac{1}{n}\sum_{i=1}^{n}|q_{xi}|+1) \tag{8}$$

The coding length for uniform quantization is:

$$\begin{aligned} r_{i0} = &-(|q_{xi}|\log_2(1-p_x)+\log_2(p_x)) - \log_2(n_0/n)+2 \\ &-(|q_{yi}|\log_2(1-p_y)+\log_2(p_y)) \end{aligned} \tag{9}$$

where n_0 is the number of vectors used for uniform quantization. The cost of uniform quantization is calculated by: $J_0 = \Sigma_i(D_0 + \lambda r_{i0})$. By setting $\partial J_0 / \partial l = 0$, we got the optimal quantization level:

$$l \approx \sqrt{6\lambda / \ln 2} \tag{10}$$

In our method, ECPNN is used to initialize the codebook. For a given bit constraint c, λ is first approximately on the rate distortion curve by (4). *"Outlier cluster"* is then created with quantization level l by (10). Residual vectors are repartitioned to the clusters with minimum cost J by:

$$Q(\boldsymbol{v}_i) = \boldsymbol{c}_j, j = \arg\min_j(J_{ij}), j = 0,1..,k \tag{11}$$

A centroid step is followed in order to update the codebook. Parameters p_x and p_y for the *geometric distribution* are also updated. Fig.3 shows an example of the codebook design. We can observe that several vectors and some clusters have completely been moved to the *"outlier cluster"*, and the size of the main codebook is reduced from 78 to 30.

The corresponding rate-distortion curve of the two-layer quantization step in Fig.2 shows better rate-distortion performance than the corresponding one-level ECPNN, or the uniform quantization under different bit-rate conditions. We should mention when the data has different property, *uniform distribution*, *negative binomial distribution* or *Poisson distribution* can also be considered instead of *geometric distribution*.

2.3. Encoding by closed-loop with dynamic programming

After prediction and quantization of the residual vectors, we compress the vector data by entropy encoding. In order to avoid error propagation, the prediction must take into account the quantization effect of the previous point by using closed-loop prediction:

$$\boldsymbol{v}_i = p_i - p_{i-1}^r \tag{12}$$

$$p_i^r = Q(\boldsymbol{v}_i) + p_{i-1}^r \tag{13}$$

here p_i^r is the approximated point after entropy coding. The total cost can be calculated by:

$$L_n = \sum_{i=1}^{n} J_i, where\ J_i = \| p_i^r - p_i \|_2^2 + \lambda r_i \tag{14}$$

where r_i is the coding length for p_i.

Selecting the quantized vector according to (13) cannot guarantee optimality during minimizing the cost function in this encoding procedure. In our method, we keep more possible candidates (t=8 in our implementation) in each step and the optimal solution is found by a dynamic programming process in the state space of size $n{\cdot}t$. Suppose that t is the best solution recorded for encoding from p_1 to p_i, with the corresponding costs $L_{i,1}, L_{i,2}, \ldots L_{i,t}$, $p_{i,1}^r, p_{i,2}^r, \ldots, p_{i,t}^r$ is the approximating points for p_i. Based on a combination of k quantized vector and t best solutions for p_i, $k{\cdot}t$ solutions are tested for approximating p_{i+1} and t best solutions $p_{i+1,1}^r, p_{i+1,2}^r, \ldots, p_{i+1,t}^r$ is saved with minimum costs $L_{i+1,1}, L_{i+1,2}, \ldots L_{i+1,t}$. In the end, backtracking is done to find the quantized vectors from $p_{n,1}^r$ with minimum cost $L_{n,1}$. The time complexity of proposed approach is O($ktn{\cdot}\log kt$).

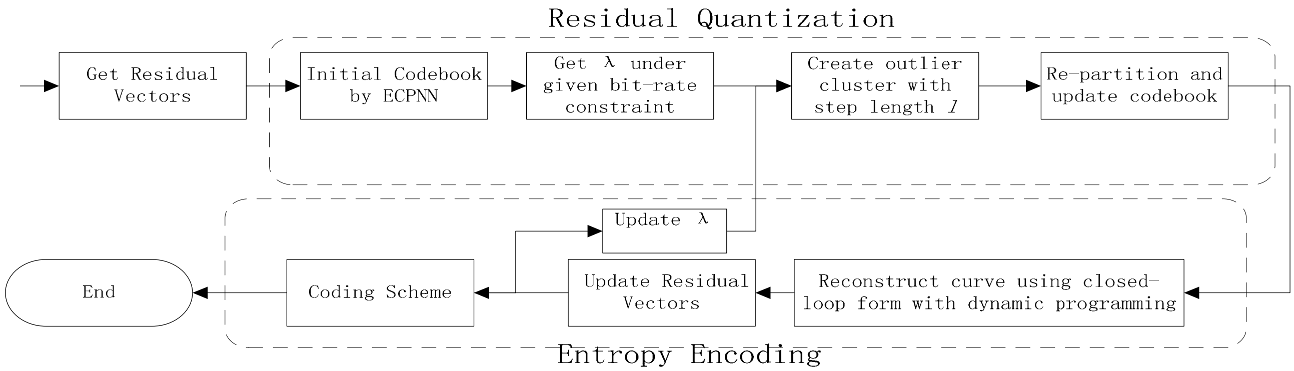

Fig. 4 Workflow of the proposed method

The residual vectors can be updated by (12) after the approximated curve has been constructed. Given bit-rate constraint c, λ is updated by a binary search in the next iteration.

3. Experiments

We evaluated the proposed algorithm with optimal codebook design (OCVQ) on a 10,911-point vector map representing the contour of Britain (Fig. 1). For comparison, two alternative methods are investigated in the experimental tests: *Clustering-based method* (CBC) [4] and the *reference line method* (RL) [5]. The distortion is measured by mean squared error (MSE). We further integrate our method into *Dynamic quantization* (DQ) [6], where integral square error (ISE) is used as the error measure. The corresponding rate-distortion curves are plotted in Fig. 5. The proposed algorithm compares favorable with the existing approach.

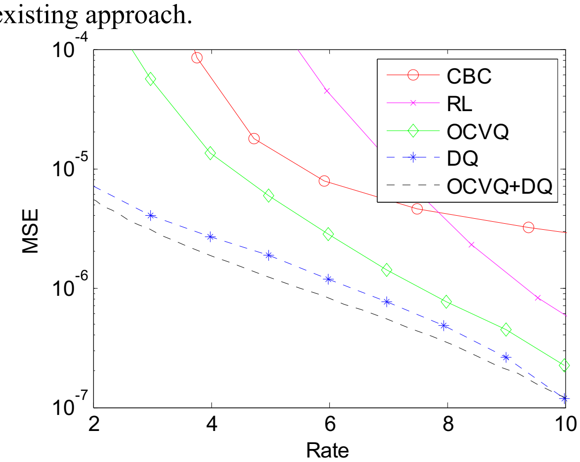

Fig. 5, Performance comparison

4. Conclusion

We propose a lossy compression algorithm for vector map data under a certain bit-rate constraint. In a comparison to the previous clustering-based method, a two-level strategy has been exploited and employed to optimize the codebook design. Vector quantization codebook is designed only for most common vectors, and the remaining vectors (outliers) are coded by additional bits using uniform quantization. Additionally, a dynamic programming method is utilized to improve the quantized vector selection in closed-loop framework, instead of using a conventionally greedy approach.

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