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Topological evolution of virtual social networks by modeling social activities

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HIGHLIGHTS

- Two mathematical abstract concepts of social activities are formalized.
- The social actions of hobbies searching and friend recommendation are characterized.
- A social activities based topology evolution model is developed.
- The model has embraced key network topological properties of real social networks.

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ABSTRACT

With the development of Internet and wireless communication, virtual social networks are becoming increasingly important in the formation of nowadays' social communities. Topological evolution model is foundational and critical for social network related researches. Up to present most of the related research experiments are carried out on artificial networks, however, a study of incorporating the actual social activities into the network topology model is ignored. This paper first formalizes two mathematical abstract concepts of hobbies search and friend recommendation to model the social actions people exhibit. Then a social activities based topology evolution simulation model is developed to satisfy some well-known properties that have been discovered in real-world social networks. Empirical results show that the proposed topology evolution model has embraced several key network topological properties of concern, which can be envisioned as signatures of real social networks.

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1. Introduction

As the Internet evolved, virtual social networks, such as Facebook, MySpace and Flickr, have great influence on interpersonal relationships and reframed the social networks [1], especially by mashing up mobile communication devices. The social networks are becoming increasingly human centric [2]. In other words, the human social behaviors and activities must be carefully studied in association with such networks [2–5]. However, it is intractable to conduct rigorous studies of human centric networking and communications over a large-scale virtual social network because of the large scale, complex topology and security problems of network. In addition, it is illegal to carry out special scientific researches and experimental

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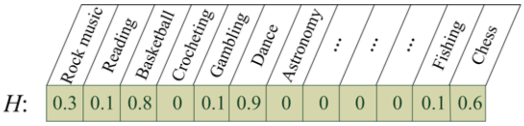


Fig. 1. Each entry $H(k) \in [0, 1]$ of the Hobbies vector denotes a hobby, such as rock music, reading, basketball and fishing.

developments on real social networks, such as social-aware routing protocol design, faults [6] and worm propagation [7–11], and advertising promotion [12]. As such, the structural modeling [13] and conceptual properties [14–16] of virtual social networks are well studied as a special form of the networks. For examples, Faloutsos et al. [17] showed that internet topologies exhibit the power law distribution. Holger Ebel et al. [18] investigated one of the earlier social network, i.e., email network, and discovered a so-called scale-free property. Boguñá et al. [19] studied a class of models of social network formation in terms of the social distance. Their model reproduces some statistical properties of real social networks, such as hierarchy of communities. Moriano and Finke [13] proposed triad formation mechanism to guarantee strong neighborhood clustering and community-level characteristics as the network size grows to infinity. A departure from the previous form of social network is the online groups or online communities which allow users to create, post, comment to and read from their own interest and niche-specific subject [20,21]. Backstrom et al. [20] studied the evolution of such online groups on real LiveJournal and DBLP social networks, and discovered that the tendency of someone to join a community is mostly influenced by the relationships with its friends. Bu et al. [22] proposed a novel evolution network model with the new concept of "last updating time", which exists in many real-life online social networks. Their model can maintain the robustness of scale-free networks and improve the network reliance against intentional attacks. Leskovec and Horvitz [23] constructed a social network model composed of 180 million nodes and 1.3 billion undirected edges based on real conversations of the MSN instant-messaging system. Their major finding is that people are tempted to interact more with each other when they have similar age, language, and location. Likewise, they discovered that a link is significantly more likely to be friendly when its two endpoints have multiple common neighbors, which means that communities are mostly formed by the principle of "the friend of my friend is my friend" [24].

It is worth noting that social groups on networks are now becoming one of footstones for human societies [25] in a way of strengthening their ties or friendship by sharing similar interests and activities etc. However, most of the current topology models lack a rigorous study over the social properties of current virtual social networks. This is because that human and social activity's impact on topological evolution model is difficult to be evaluated and hence is mostly ignored. To address this outstanding problem, an evolution model is suggested from the viewpoint of finding new friends. It characterizes the social actions of hobbies searching and friend recommendation in a social network, which are known as common ways for meeting friends and forming communities. The advantage of the underlying model is the generation of an artificial network that reproduces several key statistical properties inhabited in real social networks. It is fundamental for many social network researches including reliability estimation, worm propagation and advertising promotion.

2. Topology evolution model

Virtual social networks can be formed as the actual social relations existing in the real world through common interests and mutual acquaintances. In the networking model, each social network user can be represented by a sensor node. One pair of network users can be linked together via a network edge if there exists an interaction or association between them. The edge direction is the orientation, in which information is being transmitted, e.g., viewing someone's sharing or @ somebody. Furthermore, the weight attached to the edge reflects the frequency of the information exchanged between two nodes. The larger the weight is, the stronger relationship exists between them.

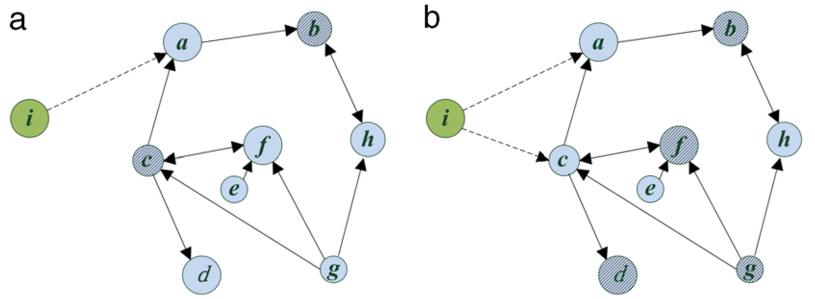
2.1. Mathematical abstract concepts

Hobbies vector: The hobbies vector H describes the interest intensity distribution of a social network user as shown in Fig. 1. The value of entries indicates the intensity of involvement or interest in the hobby. For examples, zero means that he/she is not interested in the hobby, while one denotes the maximum interest. The proposed hobbies vector can be used to seek new friends congenial to new network user and therefore is beneficial to form the community structures.

Recommendation friend set: Given a network user i is linked to the social network, a friend set RF_i can be formed by the recommendations based on its neighbor set, as Eq. (1). Fig. 2 illustrates an example of the RF_i in which the elements are dynamically updated as i links to more existing nodes. The notion $NS_i = \{j \mid w_{ij} \neq 0 \text{ or } w_{ji} \neq 0\}$ denotes neighbor set of node i, i.e., the node j is an element of set NS_i iff there exists a link between node i and node j.

$$RF_i = \bigcup_{j \in NS_i} NS_j - NS_i. \quad (1)$$

Moreover the recommendation index δ $(0 < \delta < |N|)$ is used to control the number of connected friends during topology evolution process.



(a) $NS_i = \{a\}, RF_i = \{b, c\}$.   (b) $NS_i = \{a, c\}, RF_i = \{b, d, f, g\}$.

Fig. 2. An illustration for Recommendation friend set: RF_i. (a) $NS_i = \{a\}, RF_i = \{b, c\}$; (b) $NS_i = \{a, c\}, RF_i = \{b, d, f, g\}$.

2.2. Topology generation algorithm

A directed network in the algorithm is stated as a matrix $(w_{ij})_{N\times N}$, where w_{ij} represents the weight on the directed edge from node i to node j (the weight is zero if no edges exist), with $i, j = 1 \dots N$, and N is the size of the social network. The topology generation algorithm is described as follows:

Stage 1: *Initialize the network*

The algorithm starts from an initial seed, i.e. a coupled network of N_0 nodes linked by edges with assigned weight $w = 1$. We assume that the new coming node will be connected to an existing node with weight w; this means that no isolated node exists. And the algorithm assigns an interest vector H_i to each node.

Stage 2: *Social network growing stage*

The social network is incremented in discrete time steps ($t = 0, 1, 2 \dots$), and at each step a new node is added to the network. The process of topology evolution at each time step is shown below:

(i) Select the first friend.

Given a new node i to be added to the network, a node subset sharing similar hobbies is recommended to i from existing nodes by Eq. (2).

$$Similar(i) = \left\{j | j \in N \wedge \left\|H_i - H_j\right\| < \bar{d}\right\} \tag{2}$$

where $\bar{d} = \frac{\sum_{j\in N}\|H_i - H_j\|}{|N|}$. And $\left\|H_i - H_j\right\|$ is the Euclidean distance from H_i to H_j. The smaller value of $\left\|H_i - H_j\right\|$ is, the more similar two users' profiles are. Then the algorithm randomly picks one node j from the subset $Similar(i)$ under the most likelihood or probability stated in Eq. (3). Herewith S_i denotes the strength of node i, i.e., it is the sum of weights attached to edges of node i. The underlying probability is how much interest the new user is likely to attach to the node. A directed edge $\langle i, j\rangle$ should be established with weight $w_{ij} = 1$.

$$\Pi_j = \frac{S_j}{\sum\limits_{k\in Similar(i)} S_k}. \tag{3}$$

(ii) Recommend more friends (in case of $|NS_i| < \delta$).

Let us first define a so-called branch probability as $p = 1 - \frac{1}{|RF_i|}$ ($p \in [0, 1]$, $p = 0$ in the case of $|RF_i| = 0$ and $|RF_i| = 1$), and then perform one of the operations (a) and (b) in terms of the probability p:

(a) Cascading recommendation with p.

Select one node j from RF_i under the probability given in (4), and establish a new edge $\langle i, j\rangle$.

$$\Pi_j = \frac{S_j}{\sum\limits_{k\in RF_i} S_k}. \tag{4}$$

(b) Random recommendation with $(1 - p)$.

Select a node j randomly from the subset $Similar(i)/(RF_i \cup NS_i)$ with the probability given in (5).

$$\Pi_j = \frac{S_j}{\sum\limits_{k\in Similar(i)\wedge k\notin(RF_i\cup NS_i)} S_k}. \tag{5}$$

Update the node sets $RF(i)$ and NS_i, and **repeat step (ii)** if $|NS_i| < \delta$.

Stage 3: *Dynamic evolution of weights during the growing stage*

In the stage of recommending friends, we can update the weights of directed edges between the connected node j and the node $\in NS_i \cap NS_j$ on the fly, according to formula (6) and (7). As shown in Fig. 3, the algorithm updates the weight w_{ca}

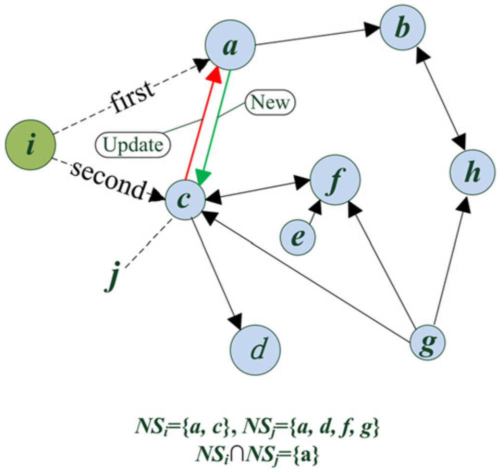


Fig. 3. Dynamic evolution of weights. $NS_i = \{a, c\}$, $NS_j = \{a, d, f, g\}$, $NS_i \cap NS_j = \{a\}$.

of $\langle c, a\rangle$ by formula (6), and adds a new edge $\langle a, c\rangle$ with weight w_{ca} by formula (7).

$$w_{jk} = w_{jk} + \frac{w_{jk} + w_{kj}}{S_j} \quad \forall k \in NS_i \cap NS_j \tag{6}$$

$$w_{kj} = w_{kj} + \frac{w_{jk} + w_{kj}}{S_k} \quad \forall k \in NS_i \cap NS_j. \tag{7}$$

3. Model discussion and analysis

In social networks, a new user is tempted to meet friends with similar interests and more activeness. This property allows us to define the user communication traffic or the user activity level as the strength of an existing given node. The larger strength the existing node is with, the more likely to be connected by the new user. It follows the principle that the new node is randomly connected to one existing node from similar hobby subsets, according to the formula (3).

In friend recommendation, parameter δ is introduced to adjust the recommendation strength. For instance, the algorithm only adds one edge in each time step and ignores the friends recommendation stage if $\delta = 1$. The parameter $\delta > 1$ means that the new node might be recommended friends by its neighbors.

During the friend recommendation process, a branch probability $p = 1 - \frac{1}{|RF_i|}$ is used for choosing operations. It follows the principle that the more nodes it is connected to, the more likely to be recommended new friends by its neighbors.

Several further design principles can be discussed as follows:

(1) First, supposed that a new added node i is linked to the node j by edge $\langle i, j\rangle$ with w_{ij}; the degree and strength of node j are increased by 1 and w_{ij} respectively.
(2) Secondly, cascading recommendation process leads to adding new edges or updating the weight of edges among node j and its neighbors. To this end, the incremental strength of node j and its neighbor k can be written as formula (8) and (9) respectively:

$$\Delta S_j = \frac{1}{S_j} \sum_{k \in NS_i \cap NS_j} w_{jk} + w_{kj} \tag{8}$$

$$\Delta S_k = \frac{w_{jk} + w_{kj}}{S_k}. \tag{9}$$

(3) Finally, the random recommendation process seems only to increase the strength of node j by 1. However, such a recommendation procedure can serve as a powerful way to connect two large-scale social communities, and thus is able to change their topology properties significantly [26].

4. Experimental results: Simulation studies by illustrative examples

To validate the proposed network topological structure, we take into account a few simplest cases of the network model. Several nontrivial properties are discussed and explained as the signatures of real social networks. In our numerical simulation, we choose the social network under the setting of $w = 1$, $N_0 = 5$ and $N = 10\,000$, where w is the initial weight, N_0 is the number of nodes of the initial coupled network, and N is the final size of the network. It also follows the assumption

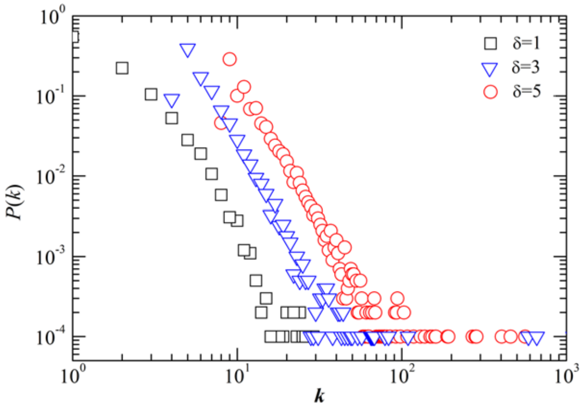


Fig. 4. Distribution of degree for different values of δ.

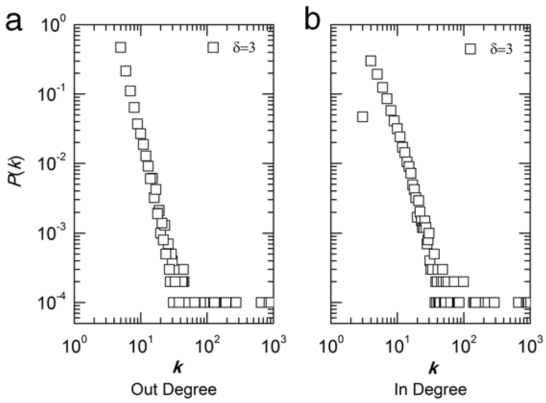


Fig. 5. Probability distribution of (a) out degree and (b) in degree with $\delta = 3$.

that the hobby obeys some kind of power law distribution [27], which means people tend to pay more attention to a few primary hobbies and spend less time on the others as shown in Fig. 1. The length of the hobby $|H|$ is set to 10.

4.1. Degree and strength

In nature, many social activities obey the property of power-law distribution [28], where the occurrence of minority events is much more than that of majority events. The power-law distribution of the network topology was discovered by the Faloutsos brothers [17] in 1999. It follows that the disparities between node degrees are significantly huge but follow a straight line with a slope of negative in the double logarithmic coordinates. The underlying linearity is the simplest way to determine if the random variables of the given instance satisfy the power-law distribution. Therefore our experiments are carried out to validate the power-law properties of the proposed model whereas different values of parameter δ are assigned to test whether the friend recommendation mechanism could affect the topology structure. As shown in Fig. 4, the degree distributions of the artificial networks always comply with the power-law with exponent $-2.46(\delta = 1)$, $-2.70(\delta = 3)$, $-2.73(\delta = 5)$ respectively.

For an insight of the proposed network model, we calculate the out degree, in degree, out strength and in strength with $\delta = 3$ respectively. The out degree and in degree distributions of nodes obtained by the proposed model are illustrated in Fig. 5. The strength distributions of nodes in the double logarithmic coordinates are shown in Fig. 6, where the node strength distribution also meets power-law property.

It turns out from our experimental results that the network topology structure generated by the proposed model is consistent with topological characteristics of real networks [17,18,29,30].

4.2. Dependence of strength on degree

Since the values of degree and strength of nodes both obey power-law distribution, we will shed a light of the relationships inherited between degree and strength. Barrat et al. [31] studied the dependence of strength S_i on degree

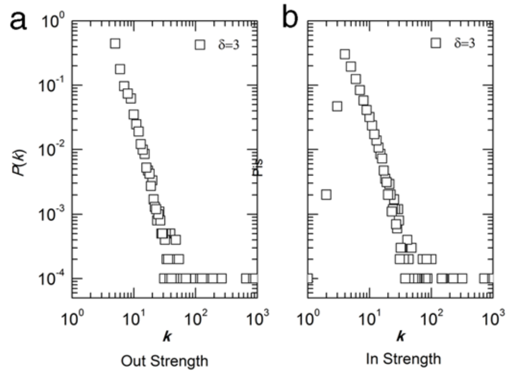


Fig. 6. Probability distribution of (a) out strength and (b) in strength with $\delta = 3$.

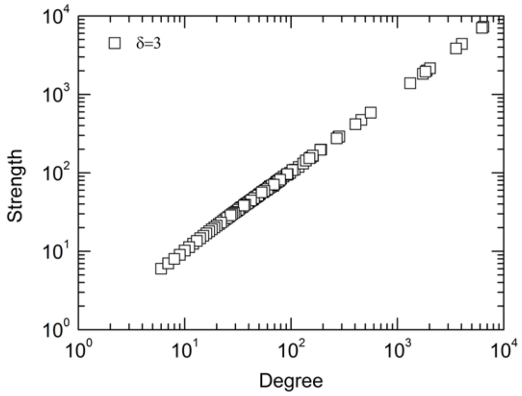


Fig. 7. Dependence of strength on degree with $\delta = 3$.

D_i, and found that the average strength $s(k)$ increases with the degree k by formula (10).

$$s(k) \approx k^{\beta}. \tag{10}$$

By analyzing a large-scale of data in social networks, they stated that the social networks follow a power-law behavior with exponent $\beta = 1.5 \pm 0.1$. Likewise, we validate the relationship between degree and strength of the artificial social network generated by the proposed model. Fig. 7 shows that it follows a power-law behavior with exponent $\beta = 1.43$, which is consistent with the empirical studies on social networks.

4.3. Aggregation

The distributions of degree and strength are insufficient when investigating the topological structure of social networks, i.e., networks with the same power-law distribution may have very different topology structures. Empirical studies demonstrate that the network nodes tend to create tightly knit groups in social networks with a higher density of ties. In addition, aggregation has attracted a considerable attention to formulate the network topology properties [32,33].

Clustering coefficient is defined as the closeness between nodes and their neighbors, which is described as follow:

$$c_i = \frac{2E_i}{k_i(k_i - 1)} \tag{11}$$

where k_i is denoted as the number of node i's neighbors, and E_i stands for the sum of edges among the neighbors of node i.

Since the clustering coefficient is the closeness of the node with its neighbors, an interaction is supposed to exist between two nodes if there is an edge regardless of its direction. In this work the clustering coefficient of nodes is calculated by transforming the directed network into undirected. Fig. 8 demonstrates that the relationship between the clustering coefficient and degree obeys a power-law behavior. The resulting clustering coefficient of the network is 0.3452 which is aligned with the statistical result reported by Mislove et al. [34].

4.4. Coreness–hierarchical structure

Social network topology structure also has the properties of being spontaneous and multi hierarchical. The hierarchy of the generated network topology can be described by using the coreness. The k-core is defined as the subgraph obtained

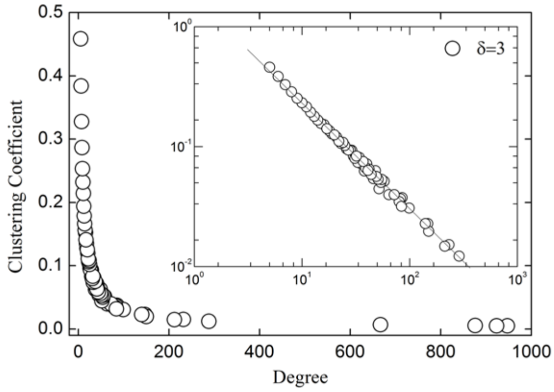


Fig. 8. Relationship between the clustering coefficient and degree with $\delta = 3$.

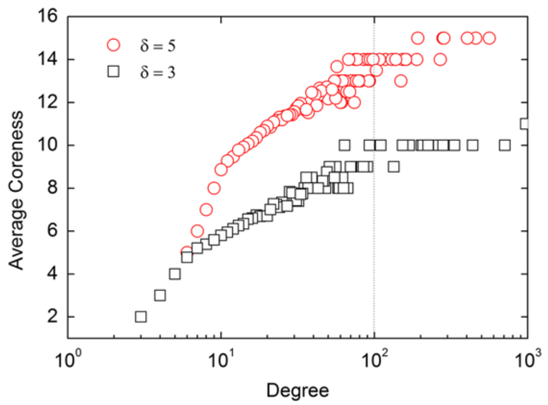


Fig. 9. Relationship between average coreness and degree with $\delta = 3$ and $\delta = 5$ respectively.

from the original graph by iteratively removing all nodes of degree less than or equal to k. Eventually, the coreness $\kappa(i)$ of node i is also the maximum k such that this node belongs to the k-core but does not exist in the $(k+1)$-core. In practice, the coreness of $\kappa(i)$ serves as a powerful metric of node connectivity rather than node degree. For examples, if the degree of a given node is high but with small coreness, then the node is not well connected such that one can easily isolate it by removing its less connected neighbors.

Fig. 9 shows the relationship between average coreness and degree of the nodes in the generated social network topology. It can be observed that the average coreness roughly follows power-law for degree < 100 as a function of node degree. And the coreness asymptotically converges to saturation as degree increases.

4.5. Heterogeneity

Empirical studies have shown that most of the complex networks in the real world embrace a property of being heterogeneous [35–37]. It means that different nodes may have significantly different network parameters such as node degrees. The principle leads to a number of distinct properties of the real social networks, i.e., scale-free property and small-world effect. To have a better understanding of heterogeneity, two important concepts of economics – Lorenz curve and Gini coefficient – are discussed in this work to measure the inequalities of network topology [36,38].

In the following, we will investigate the heterogeneity of the generated artificial network using the concepts of Lorenz curve and Gini coefficient in the context of complex network. A complex network can be represented as a graph with N vertices, denoted by $v_1, v_2, \ldots, v_N$. All vertices are sorted by the increasing order of degree (strength), which is denoted as $k_1 \leq k_2 \leq \cdots \leq k_N$ $(s_1 \leq s_2 \leq \cdots \leq s_N)$. For all the node $1 \leq i \leq N$, calculating $W_i = k_i / \sum_{j=1}^{N} k_j$.

The Lorenz curve of complex network is a curve in the rectangular coordinate system with the horizontal and vertical axis calculated as the cumulative percentage of node and degree (or strength) respectively, as shown in Fig. 10. The Gini coefficient G is defined as the ratio of the area A between curve OED and line OD to the area B of the triangle OCD.

$$G = \frac{A}{A+B}. \tag{12}$$

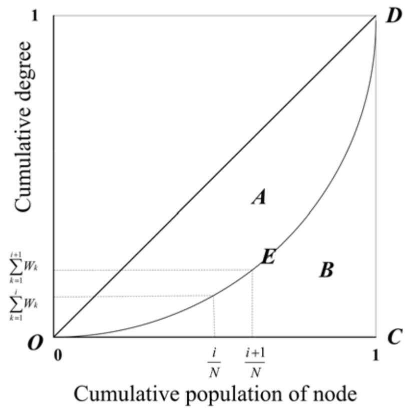


Fig. 10. Lorenz curve for complex network.

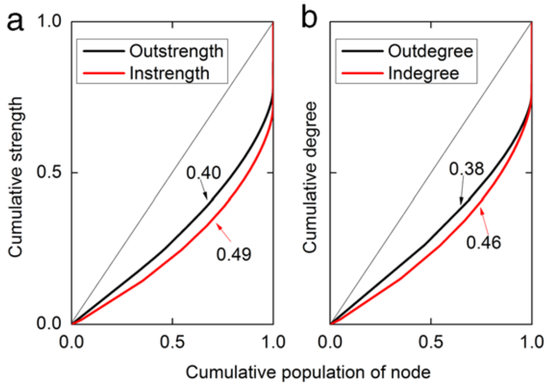


Fig. 11. Lorenz curve of (a) strength and (b) degree for the generated network with $\delta = 3$.

As well known in economics, a Gini coefficient of 0.3 or less indicates substantial equality; 0.3 to 0.4 indicates acceptable normality; and 0.4 or higher is regarded as extremely unequal [38]. We report the results of our heterogeneity analysis for the generated social network as shown in Fig. 11. The results show that Gini coefficient for in degree, out degree, in strength and out strength are 0.46, 0.38, 0.49 and 0.40 respectively. It turns out that the degree and strength distributions of the generated social network are very unequal, which is consistent with the real social networks.

5. Conclusions

Social network topology evolution model is becoming increasingly popular in many research contexts such as network reliability estimation, social-aware routing protocol design, worm propagation and advertising promotion. However, most of existing network topology models lack study of the impact of human and social behaviors in topological evolution. This paper proposed two mathematical abstract concepts of Hobbies and Recommendation to model the social activities. Then a novel topology evolution model is presented and justified in light of the two underlying schemes, namely similar hobbies searching and friend recommendation. Several simple artificial social network examples are numerically simulated using the proposed network topology model. Experiment results show that the proposed topology evolution model has identified several key network properties of concern, such as power-law distributions, dependence of strength on degree, aggregation characteristics, hierarchical structure and heterogeneity. They can be envisioned as the signatures of real social networks, and therefore they are offered as a benchmark study of virtual social networks using network topology analysis.

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