A model-based approach for aerial forest inventory

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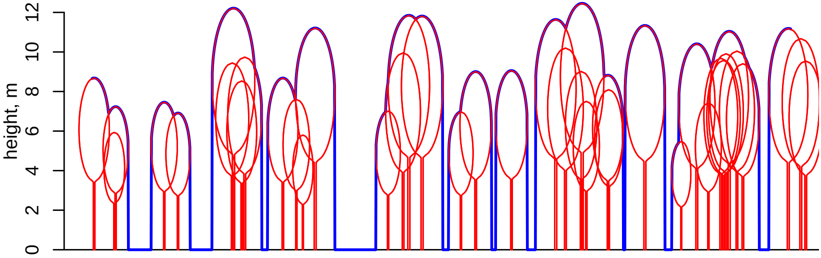
[1] University of Eastern Finland, School of Forest Sciences

March 22, 2011

Outline of the presentation

Canopy surface



Under certain simplifying assumptions (e.g., a solid top surface of a tree), we can think that

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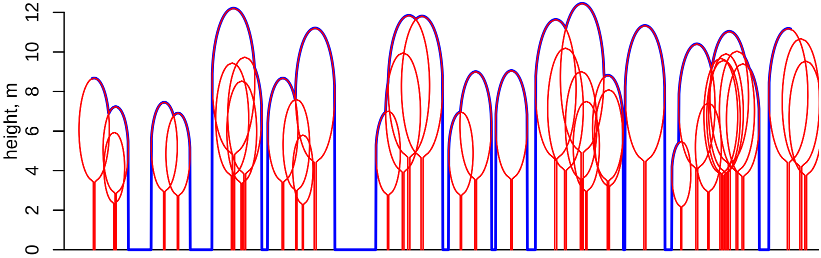

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- Individual trees generate the *canopy surface* (CS) of the stand

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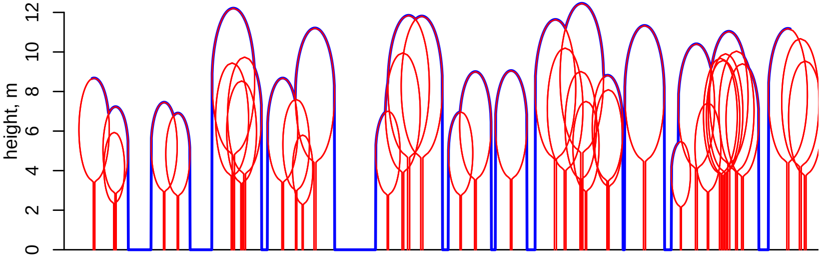
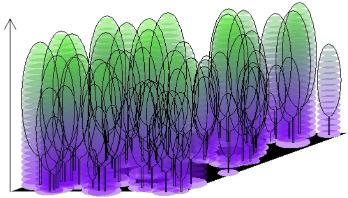
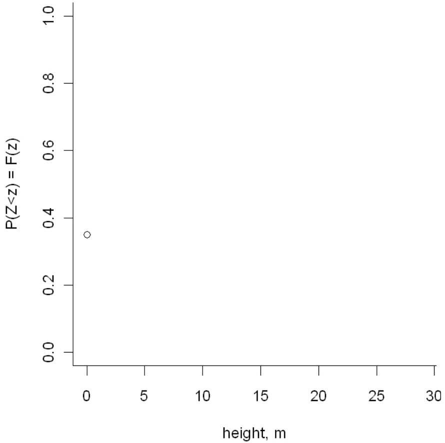


Under certain simplifying assumptions (e.g., a solid top surface of a tree), we can think that

- Individual trees generate the *canopy surface* (CS) of the stand
- ALS returns are (essentially) observations on that surface

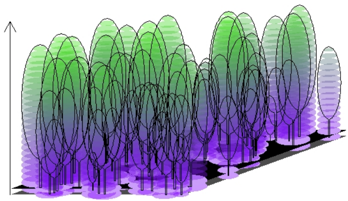
Canopy surface

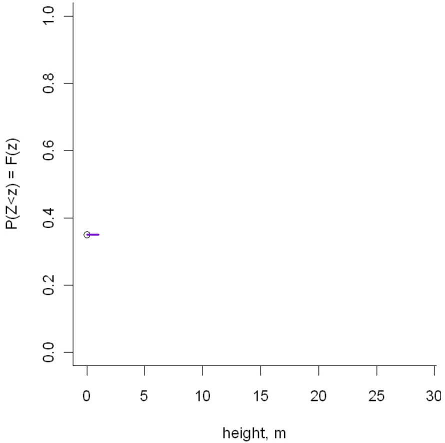




The probability to have CS below a given height
= The probability that a random point does not hit the union of tree crowns

Canopy surface





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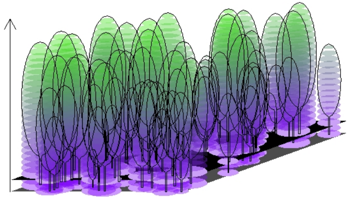

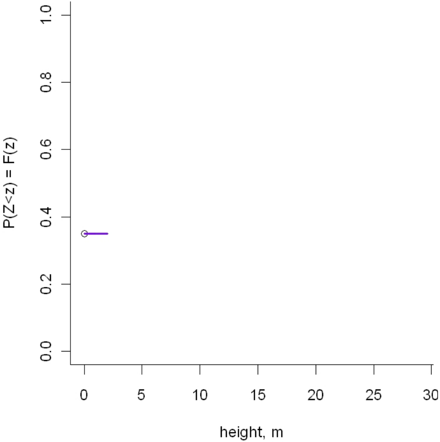

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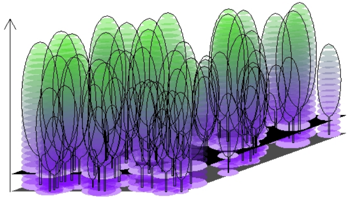

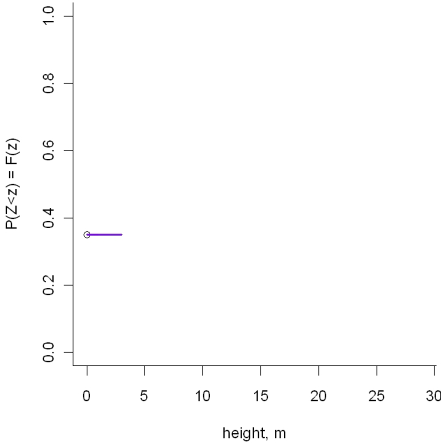

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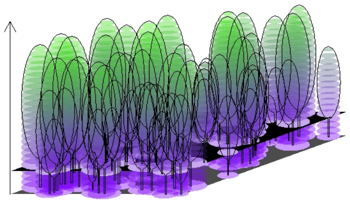

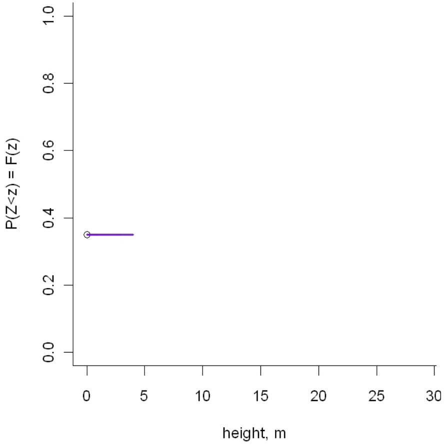

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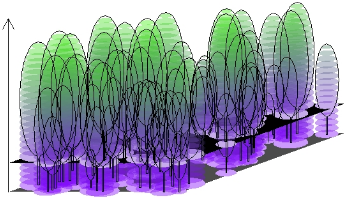

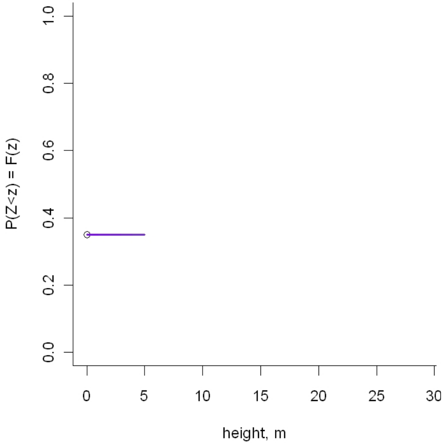

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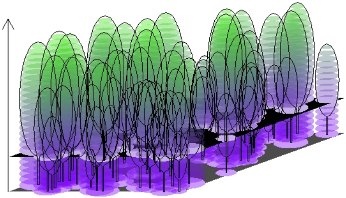

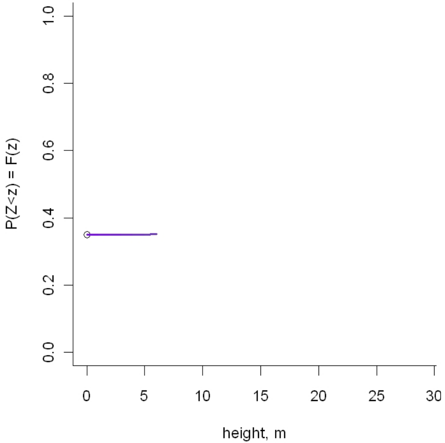

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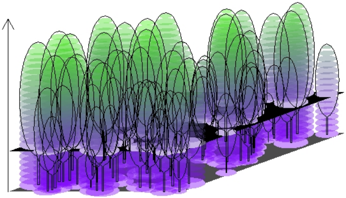

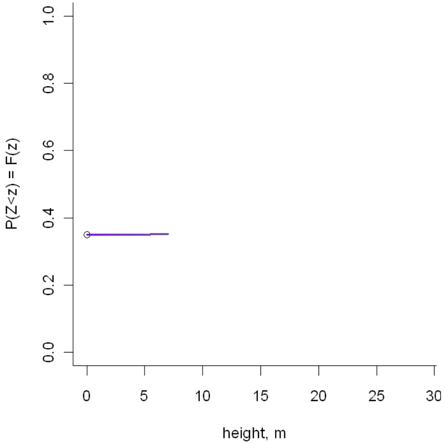

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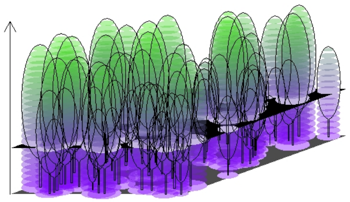

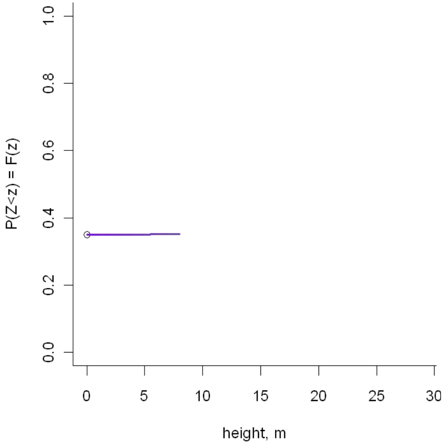

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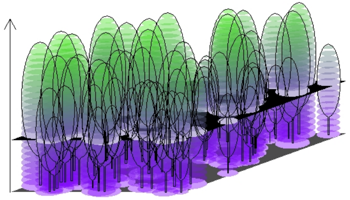

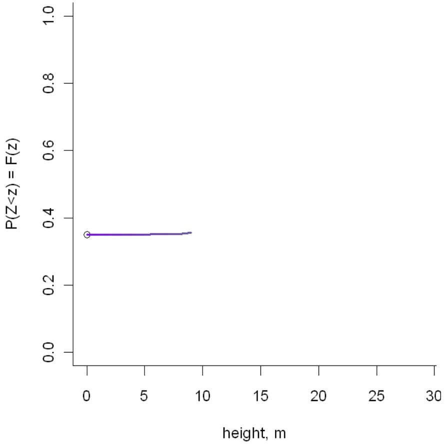

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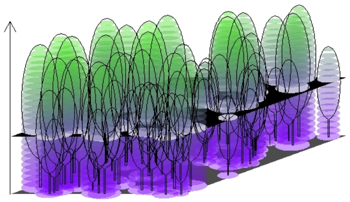

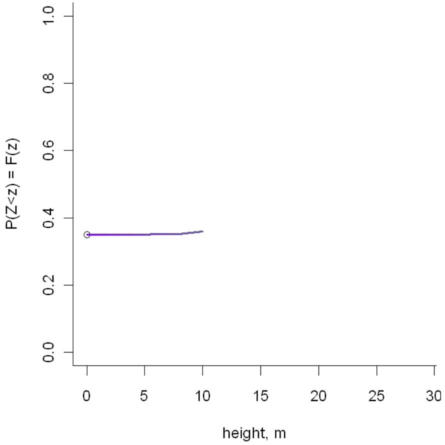

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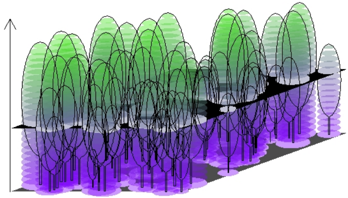

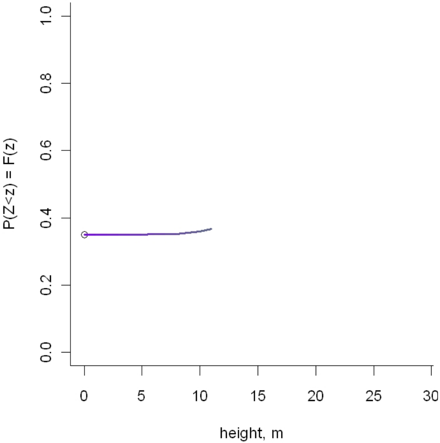

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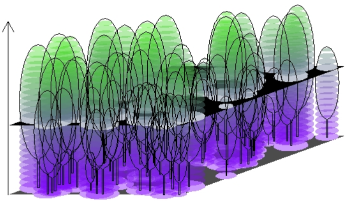

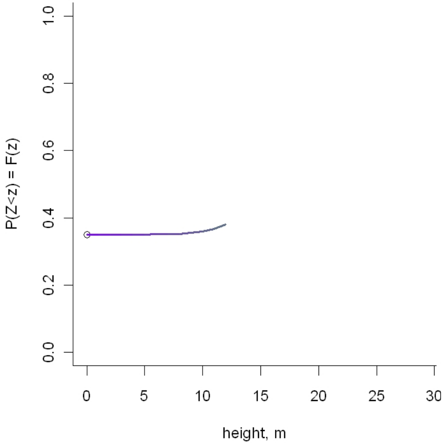

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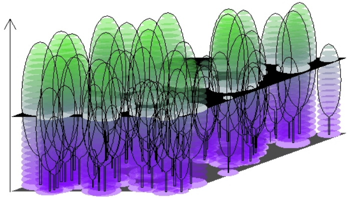

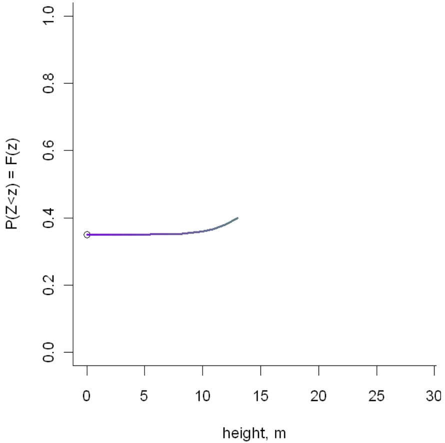

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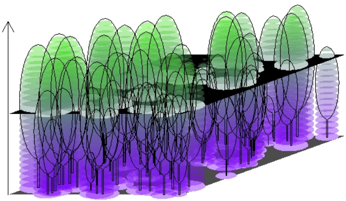

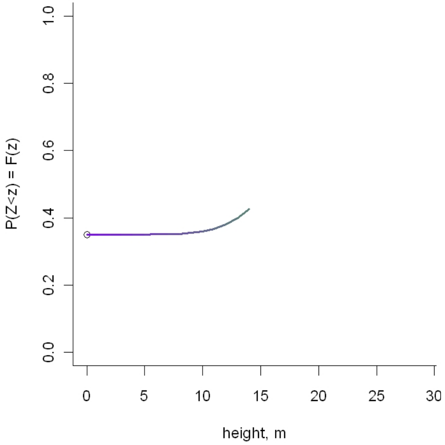

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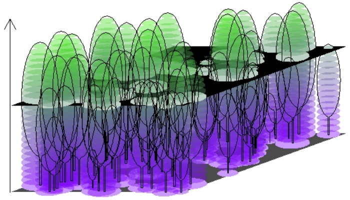

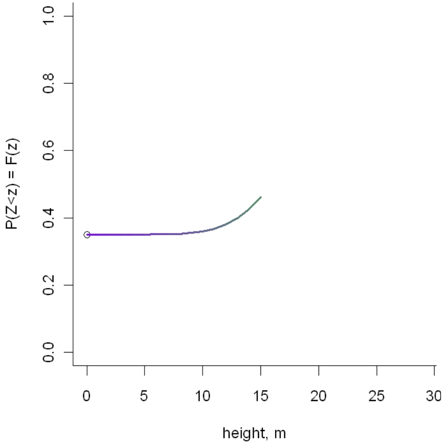

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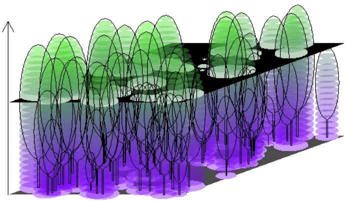

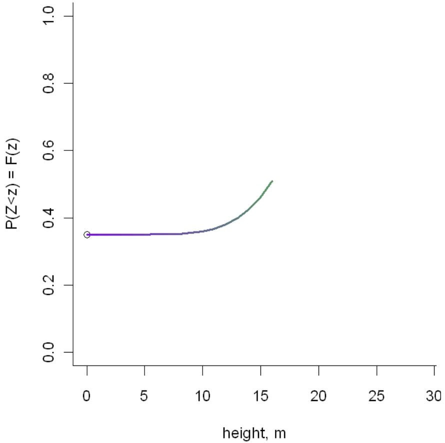

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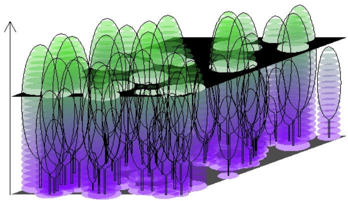

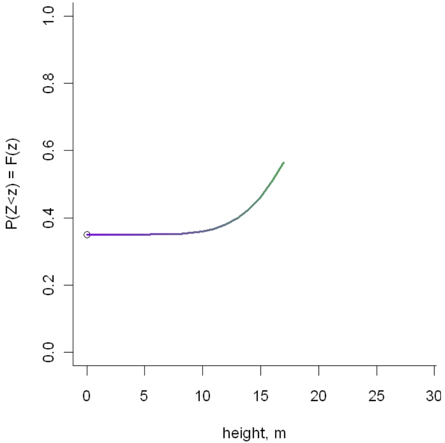

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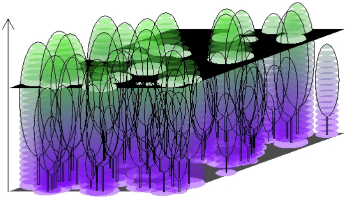

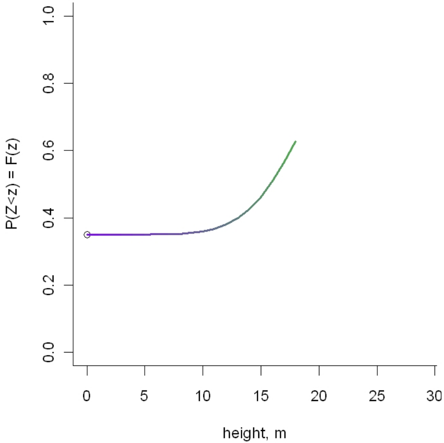

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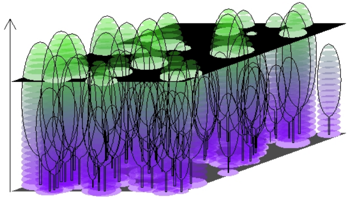

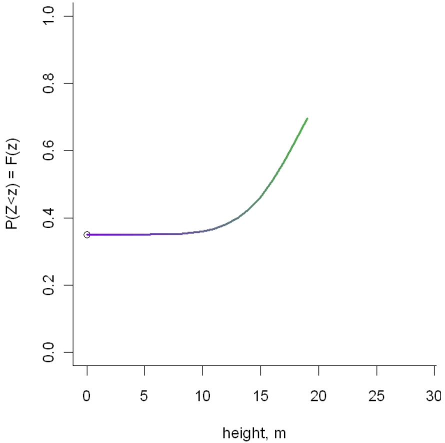

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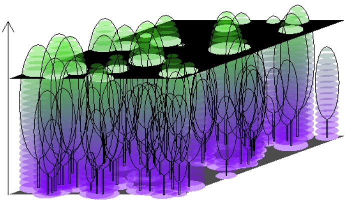

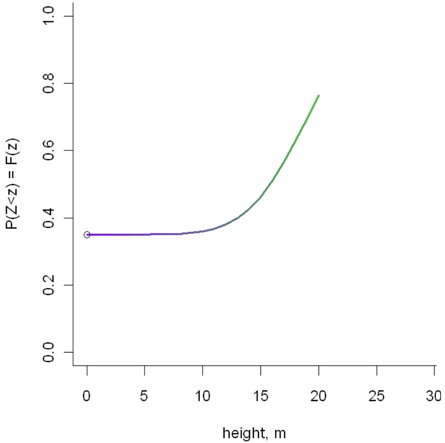

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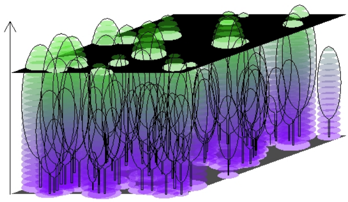

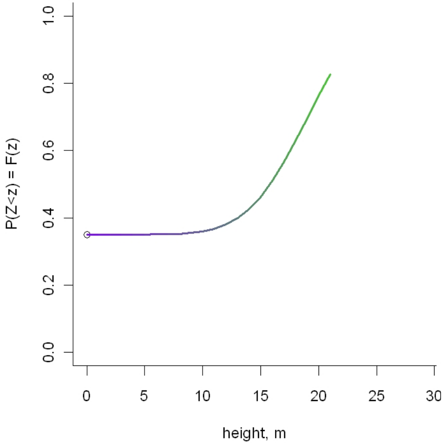

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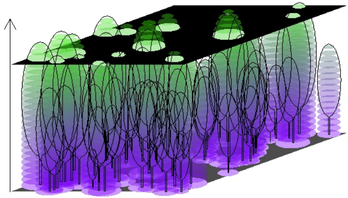

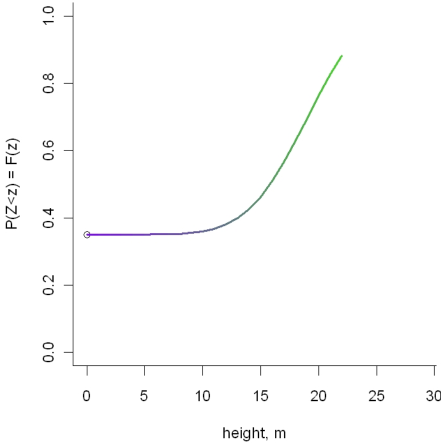

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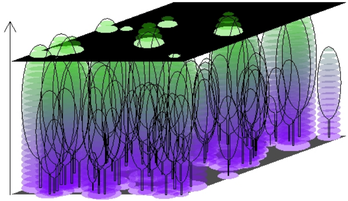

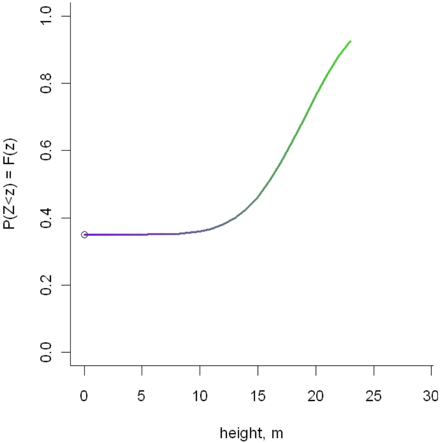

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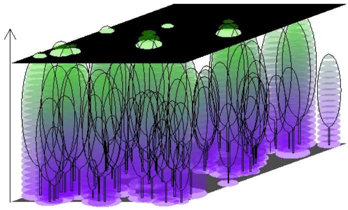

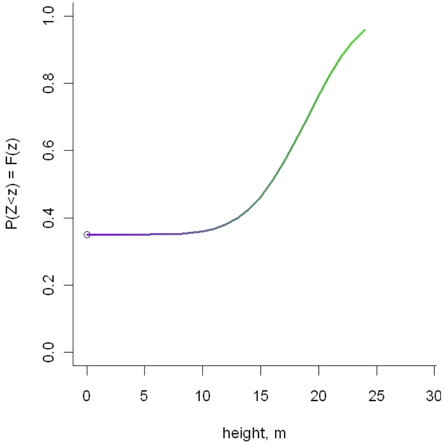

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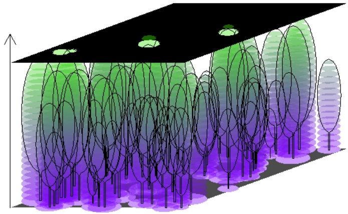

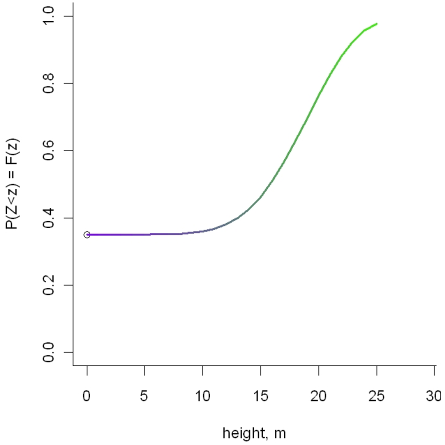

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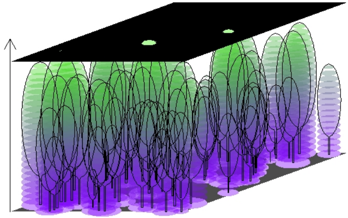

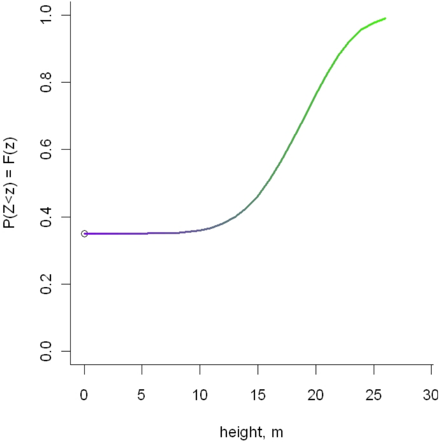

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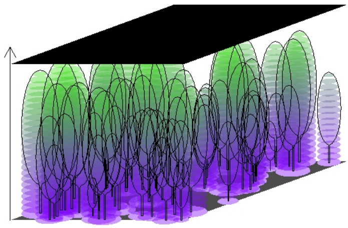
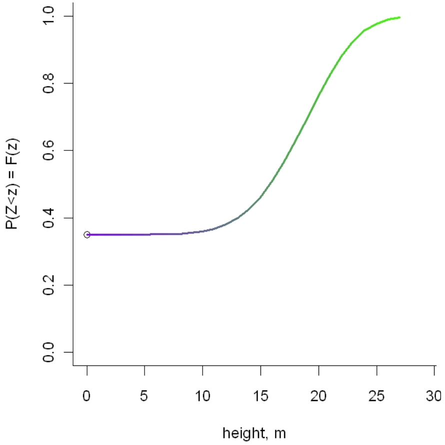




The probability to have CS below a given height
= The probability that a random point does not hit the union of tree crowns
≈ The c.d.f. of the random heights of pre-processed ALS returns, denoted by Z

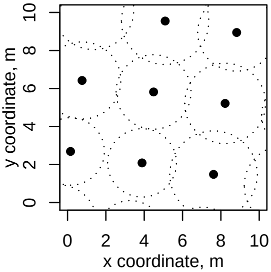
Canopy height within a stand

- We think of a forest stand as a realization of a random process.
- The canopy height at a given point is jointly specified by the following components
  1. The model that generates tree locations
  2. The model that generates tree size
  3. The model that generates the crown for a tree with given size
  4. The assumptions on how trees interact

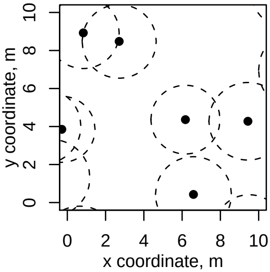
Canopy height within a stand

- We think of a forest stand as a realization of a random process.
- The canopy height at a given point is jointly specified by the following components
  1. The model that generates tree locations - random
  2. The model that generates tree size - random
  3. The model that generates the crown for a tree with given size - fixed
  4. The assumptions on how trees interact - no interaction

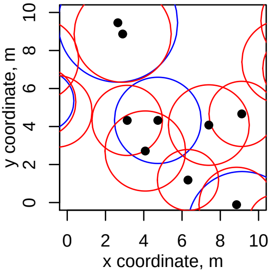
Possible models for tree locations



Square grid with random start and random orientation

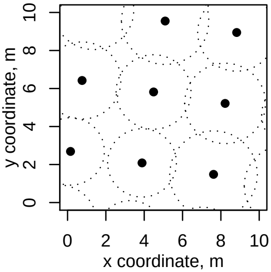


Random tree locations



Random tree locations (mixed stand)

Possible models for tree locations



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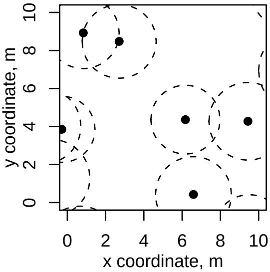

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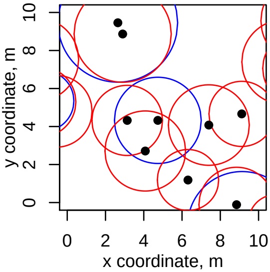
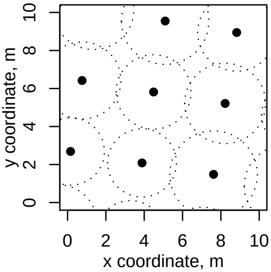


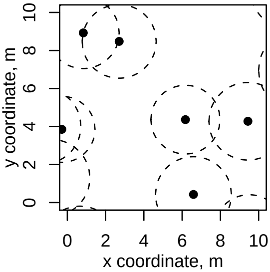
Random tree locations (mixed stand)

Parameter to be estimated: stand density λ

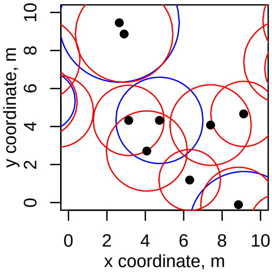
Possible models for tree locations



Square grid with random start and random orientation



Random tree locations

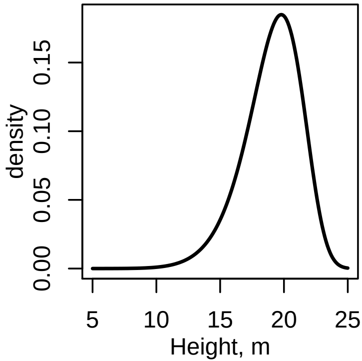


Random tree locations (mixed stand)

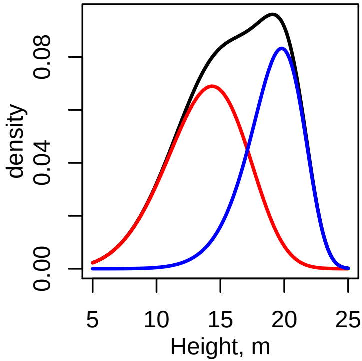
Parameter to be estimated: stand density λ

Other possibilities: Gibbs process.

Possible models for tree size

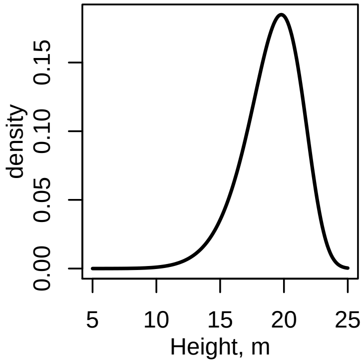


Single species stand: Weibull(α,β)

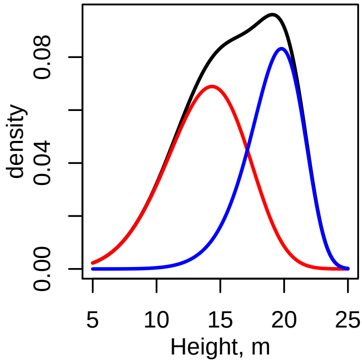


Mixed stand: Finite mixture FMWeibul(α_1,β_1,α_2,β_2,ρ)

Possible models for tree size



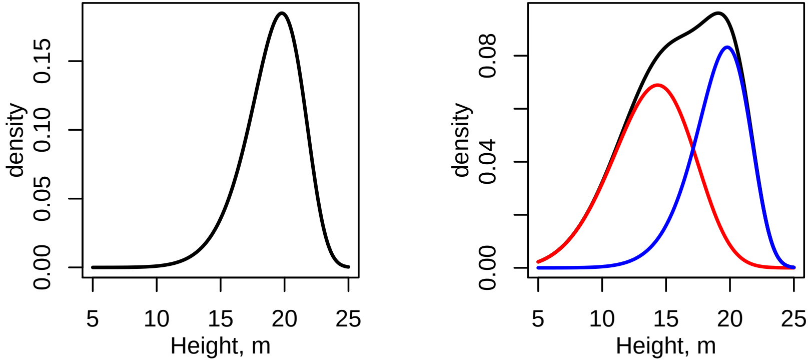
Single species stand: Weibull(α,β)



Mixed stand: Finite mixture FMWeibul(α_1,β_1,α_2,β_2,ρ)

Parameters to be estimated: α and β **or** α_1,β_1,α_2,β_2,ρ

Possible models for tree size



Single species stand: Weibull(α,β)

Mixed stand: Finite mixture FMWeibul(α_1,β_1,α_2,β_2,ρ)

Parameters to be estimated: α and β **or** α_1,β_1,α_2,β_2,ρ

Other possibilities: Normal (2 parameters), logit-logistic distribution (4 parms), Jonson's SB (4 parms).

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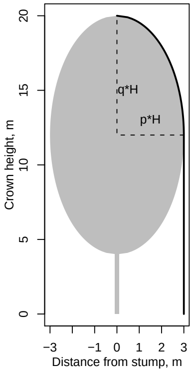
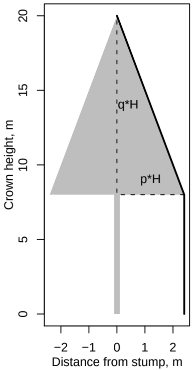
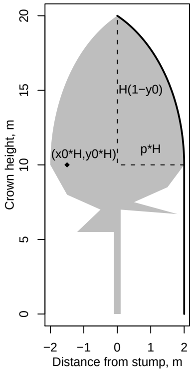
Possible models for crown shape



Simple ellipsoid

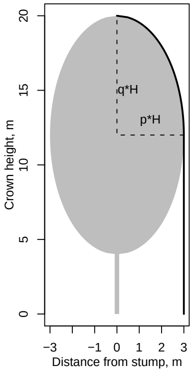


Cone

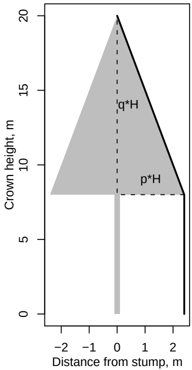


General (ellipsoid with a moving center)

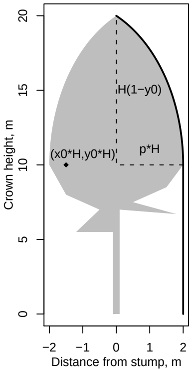
Possible models for crown shape



Simple ellipsoid



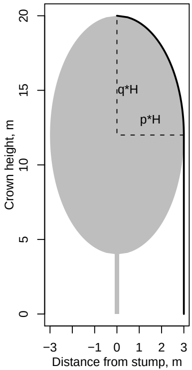
Cone



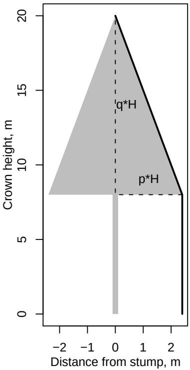
General (ellipsoid with a moving center)

Parameters p,q,x_0, and y_0 are known species-specific constants.

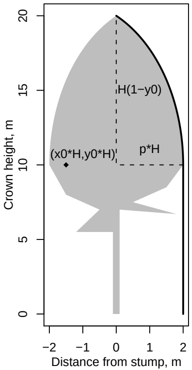
Possible models for crown shape



Simple ellipsoid



Cone

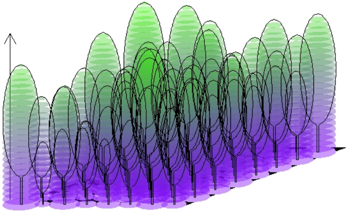


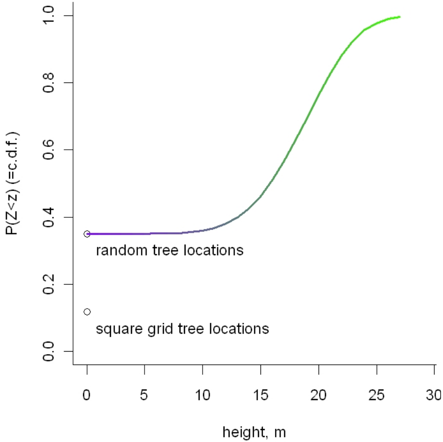
General (ellipsoid with a moving center)

Parameters p,q,x_0, and y_0 are known species-specific constants.

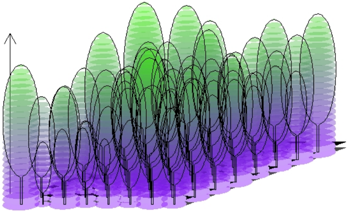
Other possibilities: Rautianen ym. 2008.

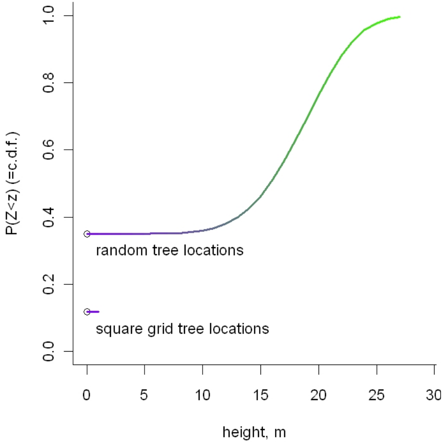
Canopy surface for square grid pattern



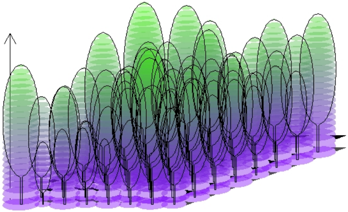


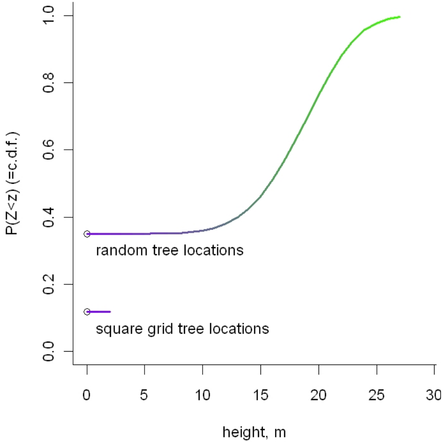
Canopy surface for square grid pattern





Canopy surface for square grid pattern





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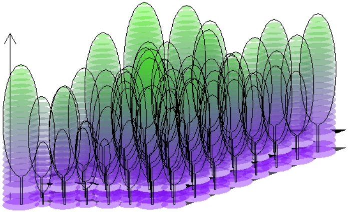

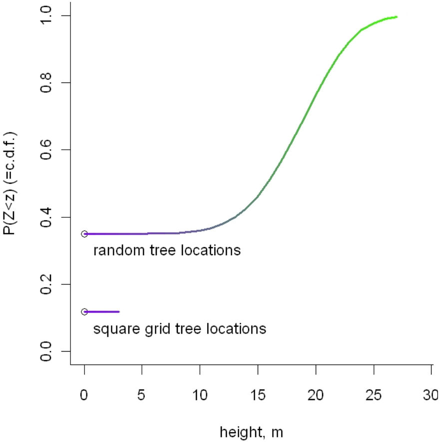

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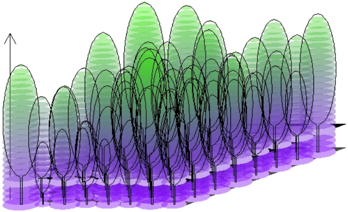
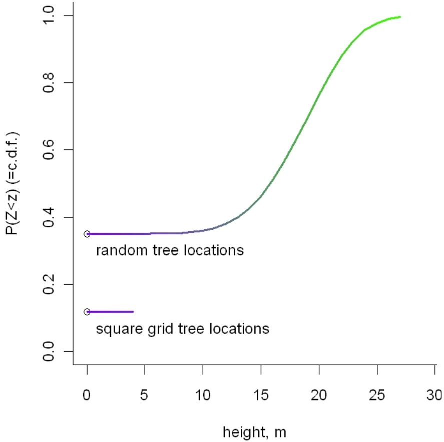
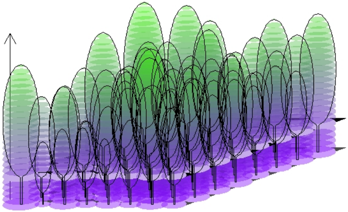
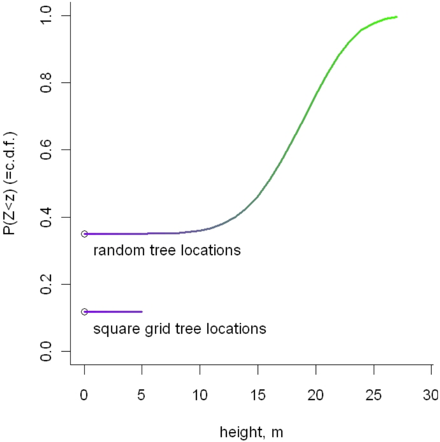




Canopy surface for square grid pattern





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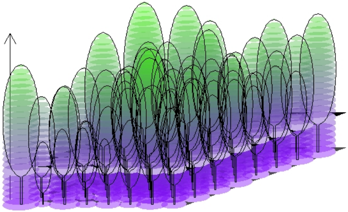

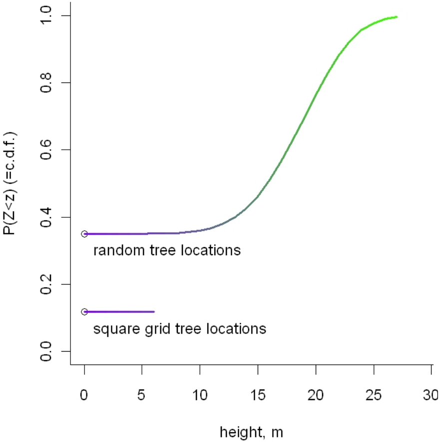

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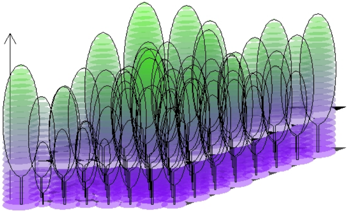

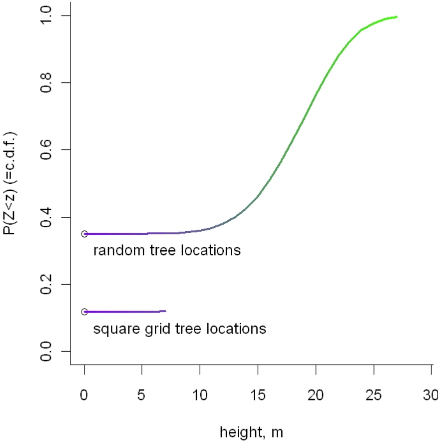

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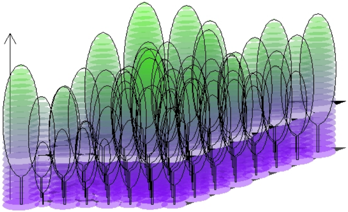

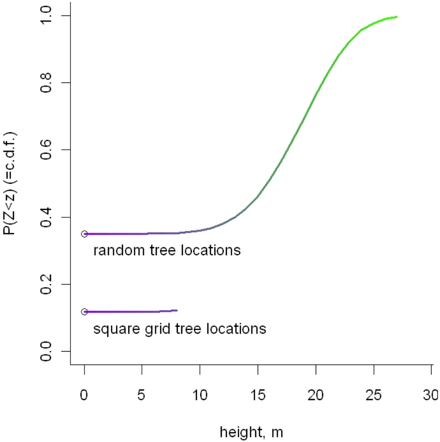

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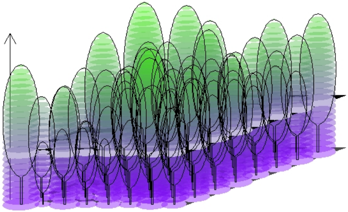

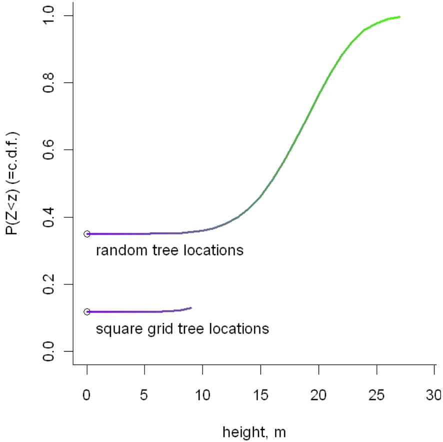

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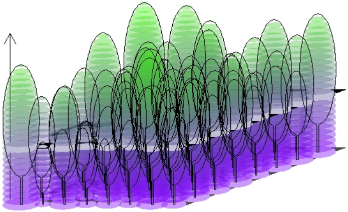

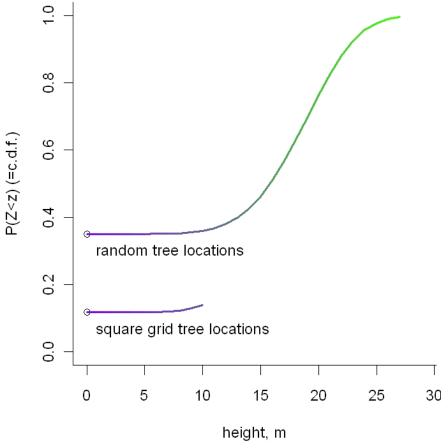

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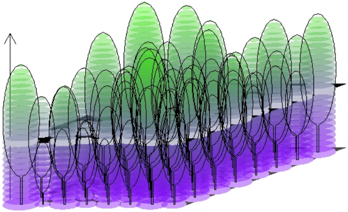

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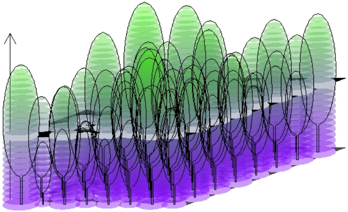

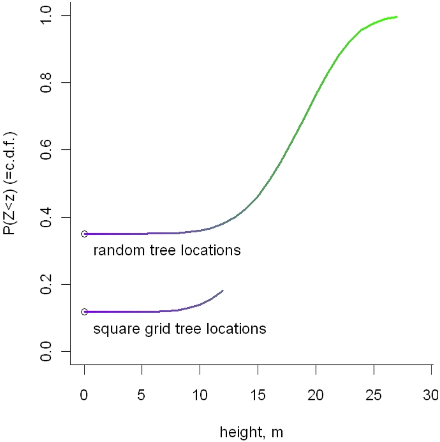

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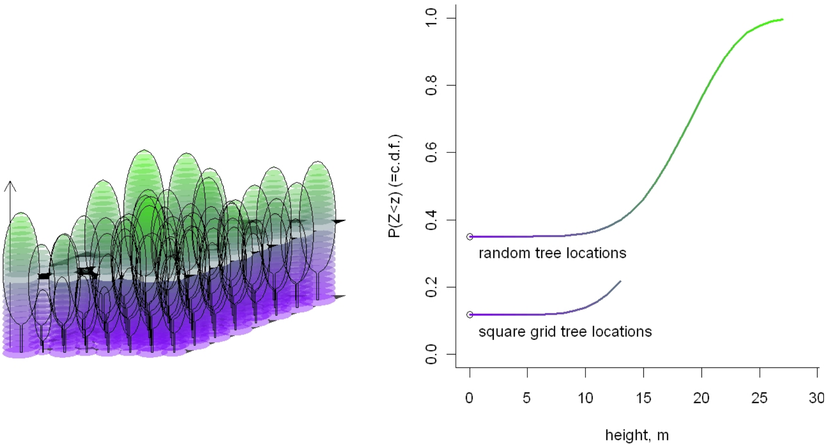

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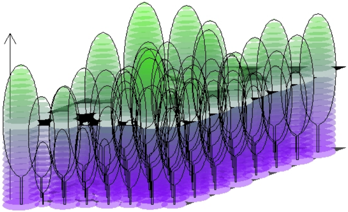

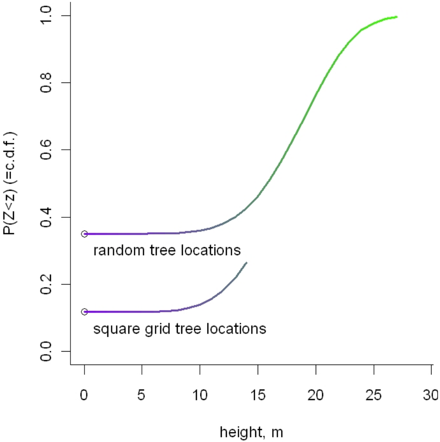

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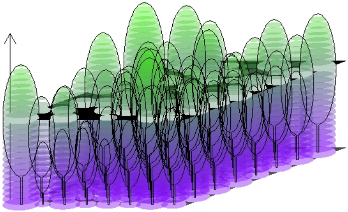

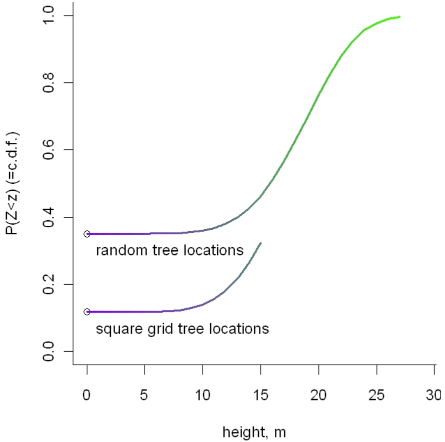

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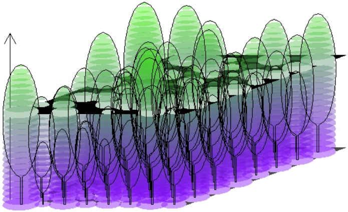

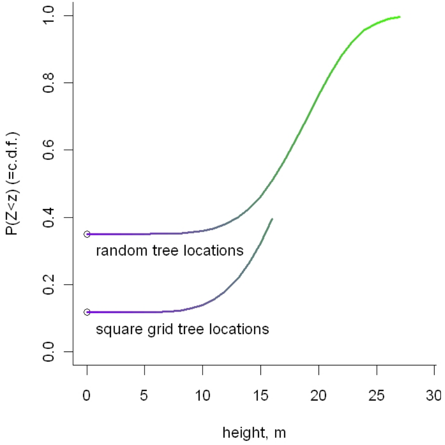

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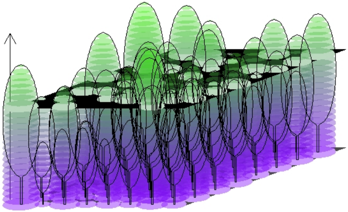

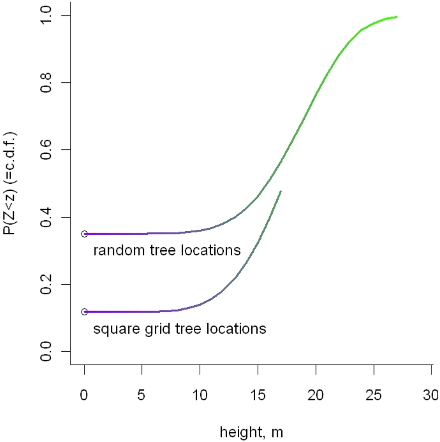

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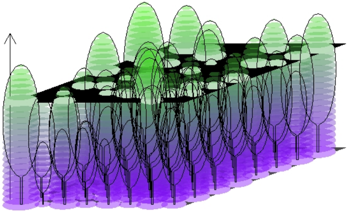

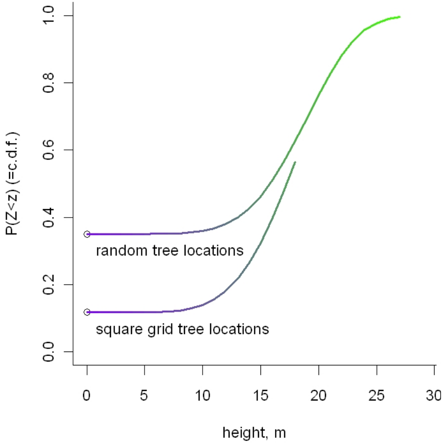

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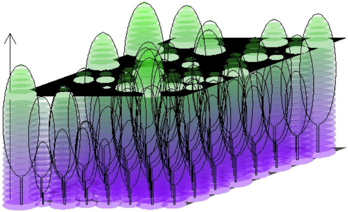

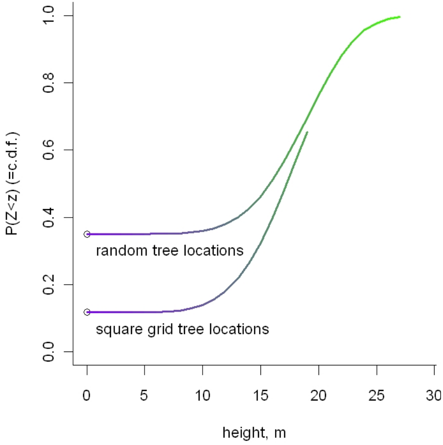

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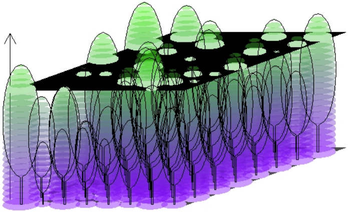

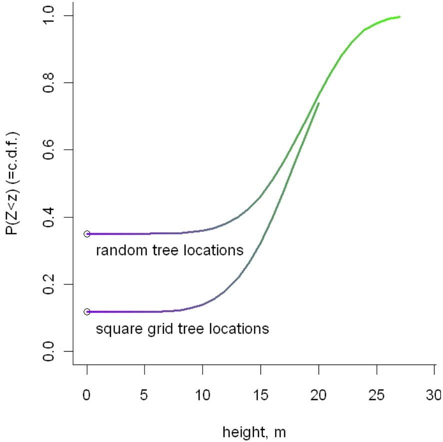

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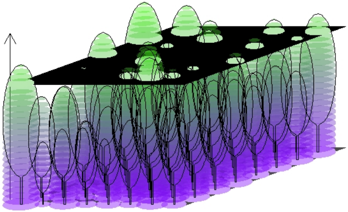

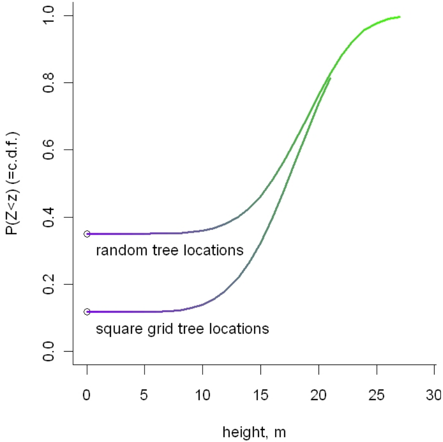

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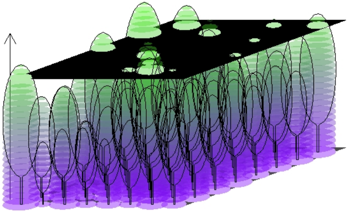
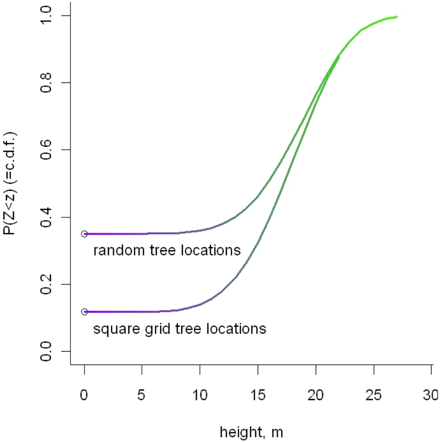
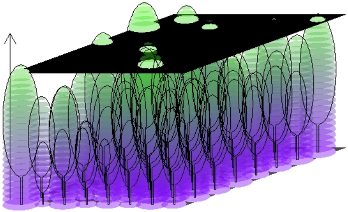
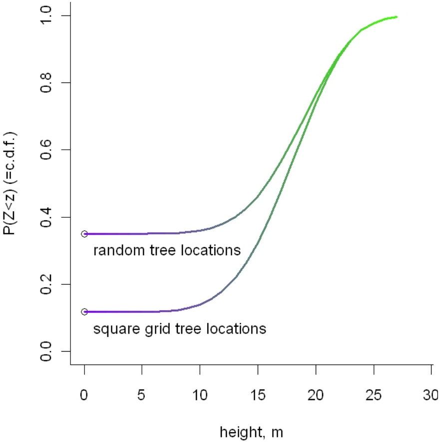
Canopy surface for square grid pattern



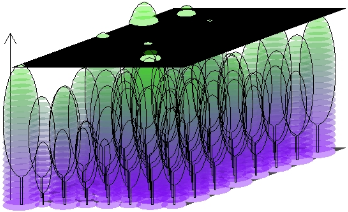


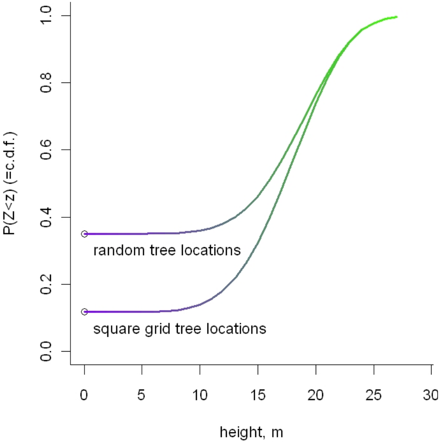
Canopy surface for square grid pattern



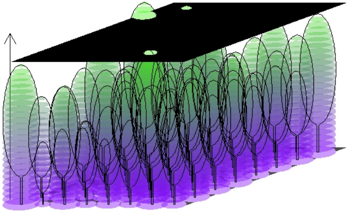


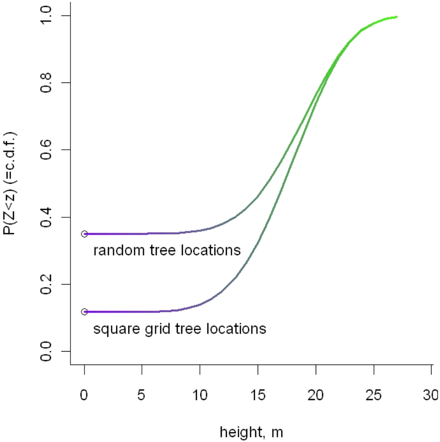
Canopy surface for square grid pattern





Canopy surface for square grid pattern





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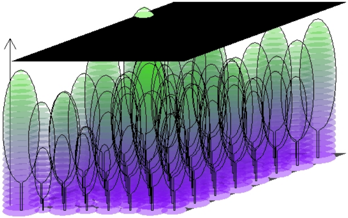

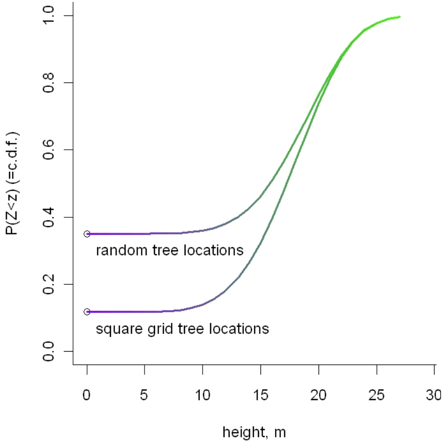

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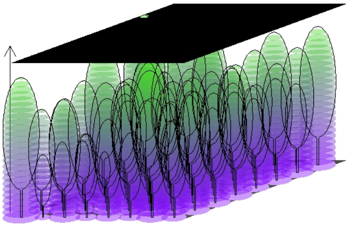
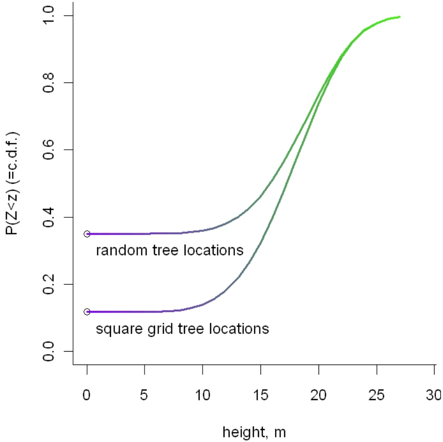
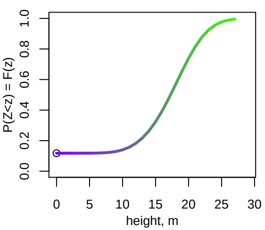
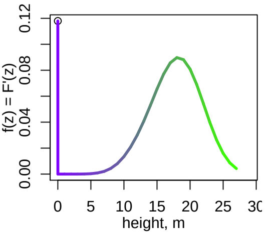




The probability density function (p.d.f.)

The p.d.f. is the first derivative of the c.d.f.





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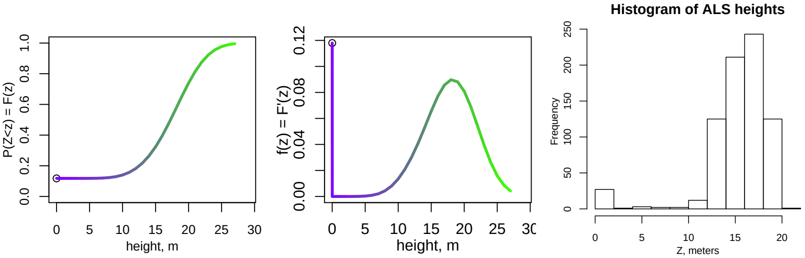

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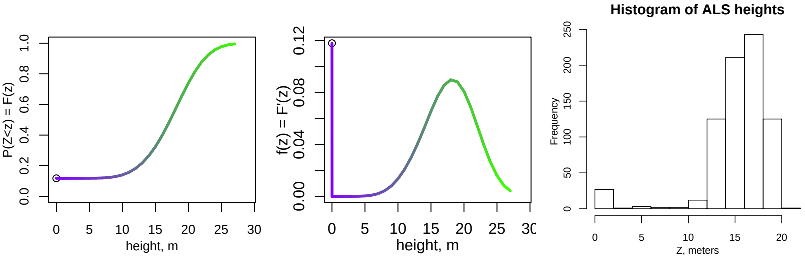

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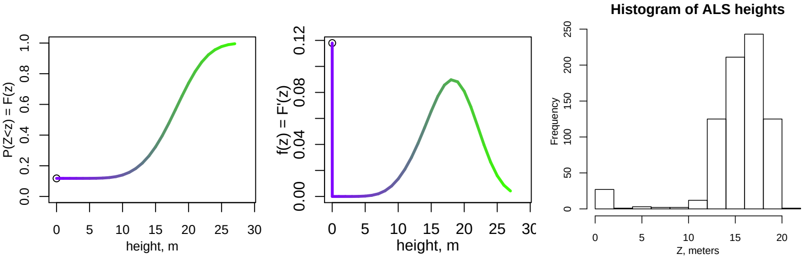

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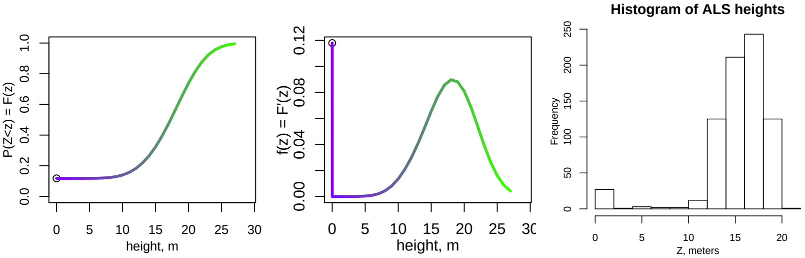

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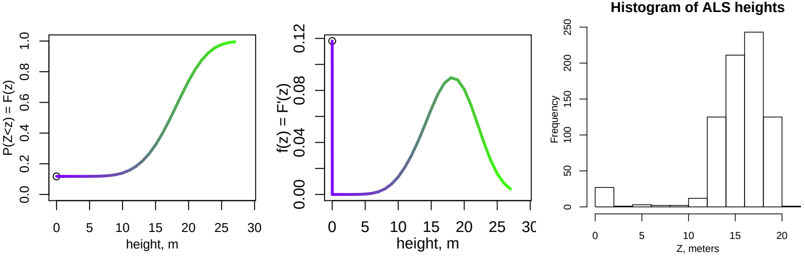

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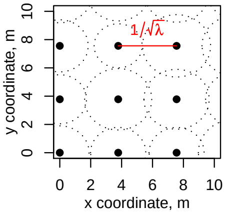
- λ is the stand density (trees per ha)

Example: Single-species forest, square grid tree locations



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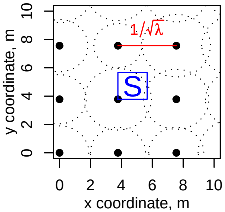

The c.d.f and density of Z become

$$G_f(z|\lambda,\boldsymbol{\theta}) = \begin{cases} 4\lambda \int_{v\in S} \prod_{i=i}^{N} F\left[Y_z^{-1}(\|u_i - v\|)|\boldsymbol{\theta}\right] \mathrm{d}v & z \geq 0 \\ 0 & z < 0 \end{cases},$$

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Single-species Poisson forest

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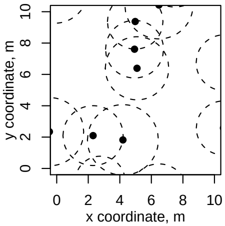


The c.d.f and density of Z become

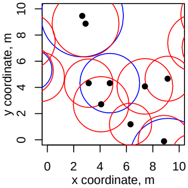
$$G_r(z|\lambda,\boldsymbol{\theta}) = \begin{cases} e^{-\lambda\pi E\left[Y(z,H_i)^2\right]} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$g_r(z|\lambda,\boldsymbol{\theta}) = \begin{cases} -\lambda\pi G(z|\lambda,\boldsymbol{\theta}) \int_z^{b^{-1}(z)} \frac{\mathrm{d}}{\mathrm{d}z}[Y(z,h)^2]f(h)\mathrm{d}h & z > 0 \\ G_r(z|\lambda,\boldsymbol{\theta}) & z = 0 \\ 0 & z < 0 \end{cases},$$

where $b(h)$ gives the height of maximum crown radius as a function of tree height.

Example: Two-species Poisson forest

Assume a mixed stand with density λ and proportions ρ and $1-\rho$ for species 1 and 2. The crown shape and distribution of tree height for species 1 are $Y_1(z,h)$ and $f_1(h|\boldsymbol{\theta}_1)$ and correspondingly for species 2.



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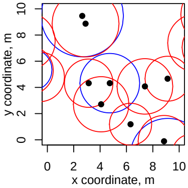

The c.d.f. and density of Z become

$$G_m(z|\lambda,\rho,\boldsymbol{\theta}) = \begin{cases} \exp\Big[-\lambda\pi\big[\rho\int_0^\infty [Y_1(z,h)^2] f_1(h|\boldsymbol{\theta}_1)\mathrm{d}h \\ \qquad\qquad +(1-\rho)\int_0^\infty [Y_2(z,h)^2] f_2(h|\boldsymbol{\theta}_2)\mathrm{d}h\big]\Big] & z \geq 0 \\ F_Z(z|\lambda,\boldsymbol{\theta}) = 0 & z < 0 \end{cases}$$

$$g_m(z|\lambda,\rho,\boldsymbol{\theta}) = \begin{cases} -\lambda\pi G_m(z|\lambda,\rho,\boldsymbol{\theta})\big[\rho I_1(z|\boldsymbol{\theta}_1) + (1-\rho) I_1(z|\boldsymbol{\theta}_2)\big] & z > 0 \\ G_m(z|\lambda,\rho,\boldsymbol{\theta}) & z = 0 \\ 0 & z < 0 \end{cases},$$

where

$$I_j(z|\boldsymbol{\theta}_\mathbf{j}) = \int_z^{b_j^{-1}(z)} \frac{\mathrm{d}}{\mathrm{d}z}[Y_j(z,h)^2] f_j(h|\boldsymbol{\theta}_j)\mathrm{d}h\,.$$

Estimation

The parameters are estimated by using the method of maximum likelihood. The log likelihood under independnce of observations is

$$\ell(\lambda, \rho, \boldsymbol{\theta}) = \sum_{j=1}^{M} I(z_j > 0) \log g(z_j|\lambda, \rho, \boldsymbol{\theta}) + M_0 \log G(0|\lambda, \rho, \boldsymbol{\theta})$$

The values of λ, ρ, and $\boldsymbol{\theta}$ that maximize the log likelihood are the ML-estimates for stand density, species proportions, and parameters of the height distribution. Asymptotic properties of ML-estimator can be used for to assess their accuracy and make inference. Other mathematically less demanding possibilities

- Method of pecentiles or moments
- Spline approximation of the pdf (Virolainen 2010)

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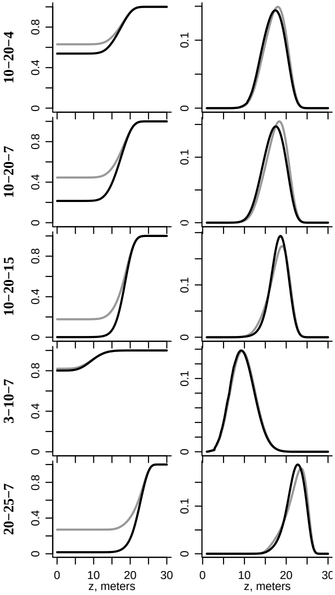
Evaluation with simulated data

Simulation results (Mehtätalo and Nyblom 2009, 2011) showed that the method finds good estimates for stand density and tree height distribution with simulated datasets where all the assumptions are met and crown shape functions are known.

The effect of spatial pattern on the distribution

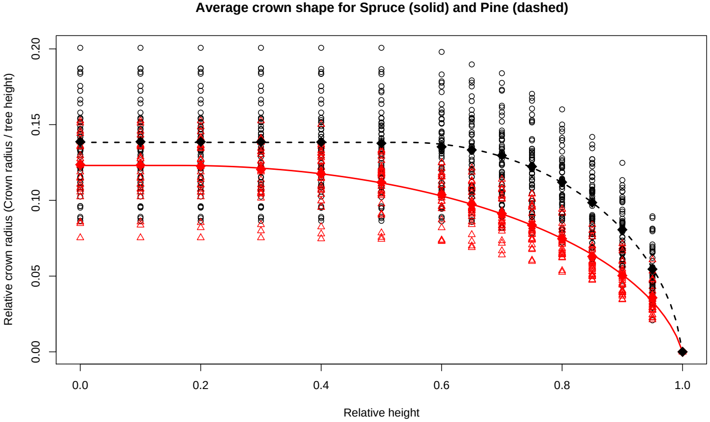


- The same values for stand density and Weibull parameters were used using
  - Square grid pattern (black), and
  - Random spatial pattern (gray)
- The graphs on the left show the c.d.f.'s of all observations
- The graphs on the right show the p.d.f.'s of canopy hits
- The values on the left show
  - shape (α) and
  - scale (β) parameters of the Weibull parameters, as well as
  - the stand density (λ, 100 trees per ha).
- The crown shape was ellipsoid with half axes $0.1H$ and $0.4H$.

Tests with real data

- Laser data 40 pulses/m^2 and aerial photographs from Tielaitos.
- Ground data (20 plots of size 20*20m). Species and dbh known for each tree.
- Single tree from 18 plots were used for modeling crown shape (the models I showed earlier)
- Thinned "laser" data (0.25 pulsesper m^2) of the rest two plots were used for method testing.
- Crown shape function was extracted from detected Norway spruce trees and applied for one pure Spruce stand to estimate stand density and tree height distribution.

Models for crown shape



$$\frac{r}{H} = \begin{cases} y_0 + b & \frac{h}{H} \leq x_0 \\ y_0 + b\sqrt{1 - \frac{(\frac{h}{H} - x_0)^2}{a^2}} & x_0 < \frac{h}{H} \leq 1 \\ 0 & \frac{h}{H} > 1 \end{cases} \quad \text{where } a = \sqrt{\frac{b^2(1-x_0)^2}{b^2 - y_0^2}}$$

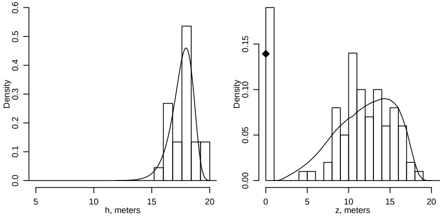
Evaluation plots

Pure spruce stand
$\bar{H}$=17.77 m
N=700 stems/ha

Mixed spruce-pine stand
$\bar{H}_{spruce}$=9.87 m
$\bar{H}_{pine}$=15.66 m
N=1350 stems/ha
ρ=0.70 (70% were spruces)

An example with a Norway spruce plot

Assuming square grid spatial pattern



$\widehat{\lambda}$=689 r/ha (true 700); $\widehat{\bar{H}}$=17.56 (true 17.77)

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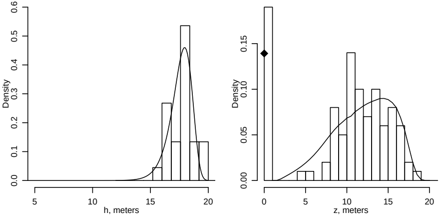

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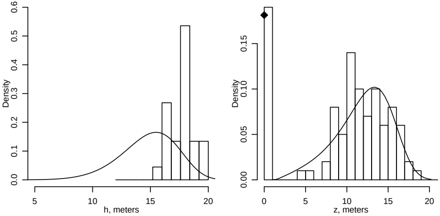
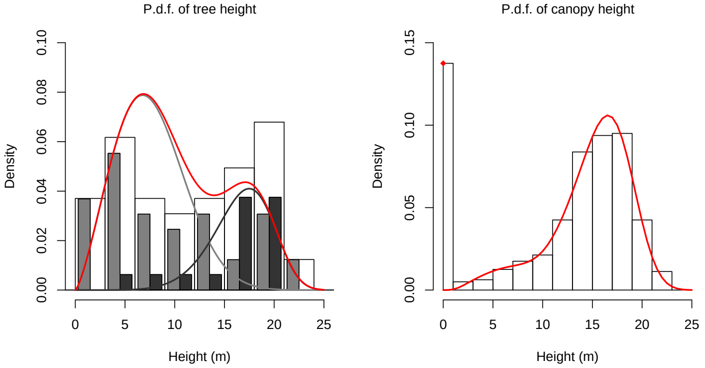
Assuming random spatial pattern



$\widehat{\lambda}$=1788 r/ha (true 700); $\widehat{\bar{H}}$=14.81 (true 17.77)

A mixed stand by assuming random locations



$\widehat{\lambda}$=2645 r/ha (true 1350)

$\widehat{\rho}$=0.6998 r/ha (true 0.7037)

$\widehat{\bar{H}_{spruce}}$=7.57 (true 9.87)

$\widehat{\bar{H}_{pine}}$=16.72 (true 15.66)

Application to area-based inventory

1. Training stage using training sample plots

2. Prediction stage using evaluation plots

Application to area-based inventory

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   1. Estimate $\xi = (\alpha, \beta)'$ by fitting Weibull distribution to the measured tree heights
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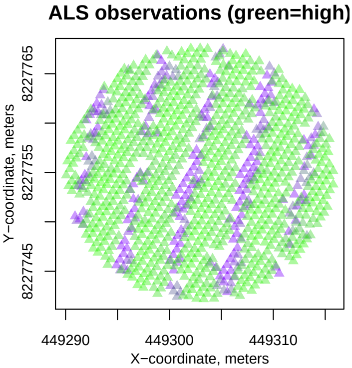
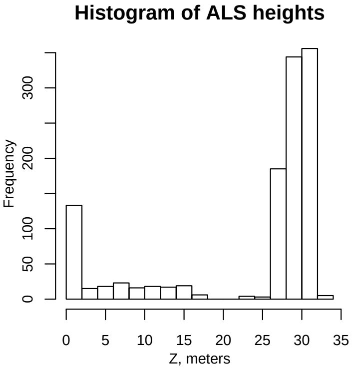
- If maximum likelihood is used in fitting, then asymptotic standard errros of estimates can be computed, too.

Study material

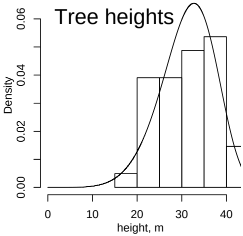
- 18 pairs of sample plots from the Veracel data (18 training and 18 evaluation plots).
- Distance between trees and stand density λ are known
- Three heights known for every 7th tree, and imputed for others using a stand-specific model
- ALS data were pre-processed and thinned to include ≈ 122 uniformly placed observations of canopy height (Z) for each plot (0.23 pulses/m^2)





Example fit

Training stage with teaching plot 11



Prediction stage with evaluation plot 11

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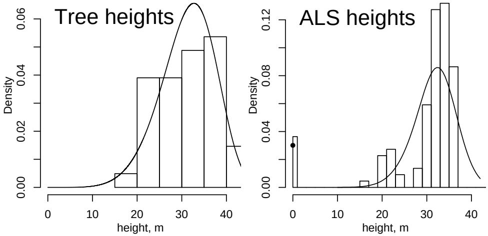

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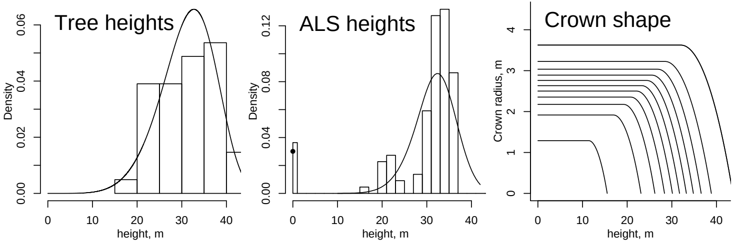

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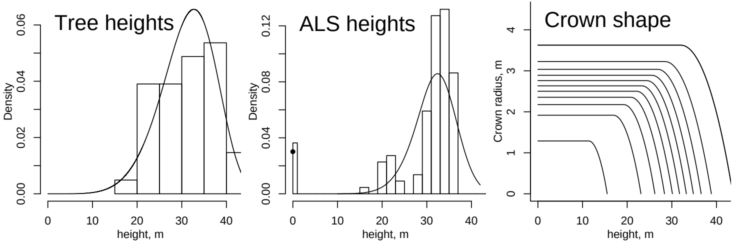

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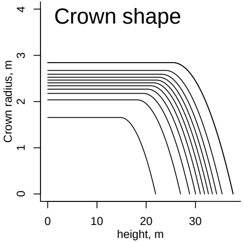

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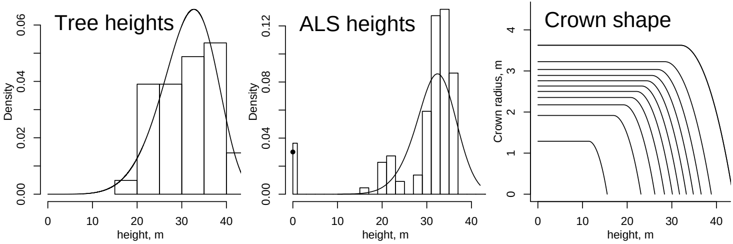

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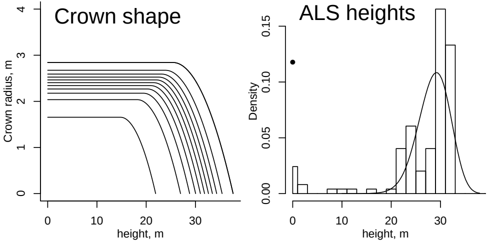

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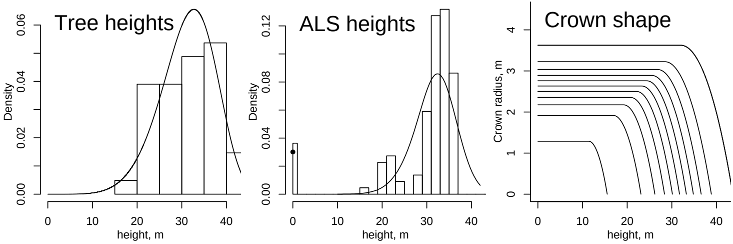

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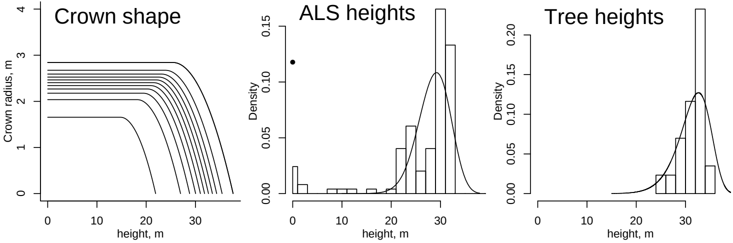
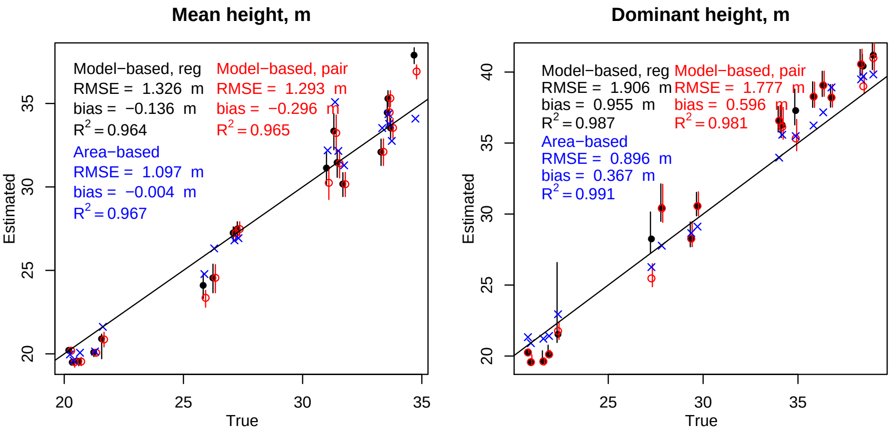


Results


Mean height, m
Model−based, reg
RMSE = 1.326 m
bias = −0.136 m
R2 = 0.964
Model−based, pair
RMSE = 1.293 m
bias = −0.296 m
R2 = 0.965
Area−based
RMSE = 1.097 m
bias = −0.004 m
R2 = 0.967
Estimated
True
20
25
30
35
Dominant height, m
Model−based, reg
RMSE = 1.906 m
bias = 0.955 m
R2 = 0.987
Model−based, pair
RMSE = 1.777 m
bias = 0.596 m
R2 = 0.981
Area−based
RMSE = 0.896 m
bias = 0.367 m
R2 = 0.991
Estimated
True
40

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- Currently, heavy computations make estimation and model development slow. Solution: spline (Virolainen and Tuomela) or percentile-based method (not yet implemented).
- The model could be generalized to many interesting situations, e.g., to mixed stands or to bivariate observations of canopy height and return intensity.

Modelling the penetration

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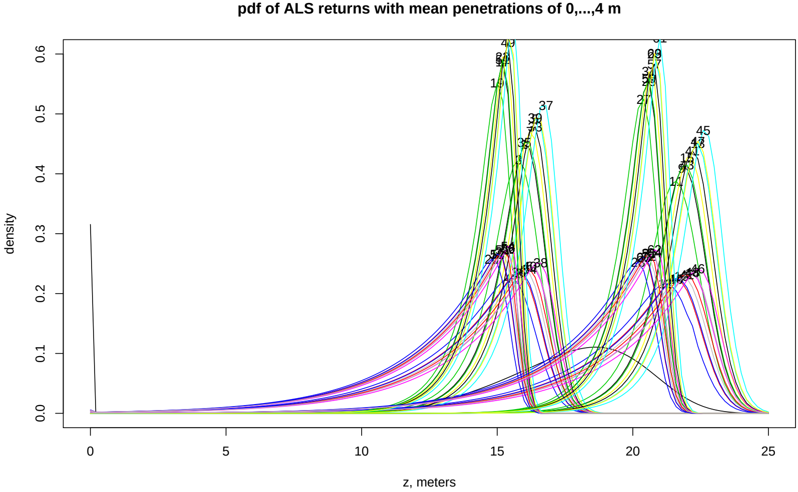
1. Ajatellaan, että jokaiseen havaintoon liittyy vakio penetraatio (esim 1 metri), jokaon estimoitava parametri.
2. Ajatellaan, että uppoama on eksponentiaalisesti jakautunut satunnaismuuttuja (Beer Lambertin laki). Silloin penetroituneiden laserkorkeuksien tiheysjakauma on
$$g_p = \begin{cases} \int_0^\infty g(z|\lambda, \boldsymbol{\xi}, \boldsymbol{\theta}) f(z-h|\rho) dh & z > 0 \\ G(z|\lambda, \rho, \boldsymbol{\theta}) + \int_0^\infty g(z|\lambda, \boldsymbol{\xi}, \boldsymbol{\theta})[1 - F(z|\rho)] dz & z = 0 \end{cases}$$
jossa g ja G ovat aiemmin johdetut jakaumat ilman uppoamaa, f ja F ovat eksponentiaalisen jakauman tiheys- ja kertymäfunktiot, ja parameri ρ kuvaa latvuston reikäisyyttä (vrt Lauri K:n esitys).

Eksponentiaalisen penetraation vaikutus jakaumaan (1 m vs 3 m)



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