Collected Papers 2020 - A Special Issue on Fermat's Last Theorem

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29th December, 2020

Chapter 1

A Special Case of Fermat's Last Theorem

A Special Case of Fermat's Last Theorem is Proved.

1.1 Introduction

Fermat's Last Theorem states that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2 [1]. We will prove a special case of this theorem: no two integers a and c other than zeros satisfy the equation $a^n + a^n = c^n$ for any integer value of n greater than 1.

1.2 A Proof

Let's assume that

$$a^n + a^n = c^n.$$

Then

$$2a^n = c^n$$

and from this we get

$$\sqrt[n]{2}^n a^n = c^n$$

and from this

$$(\sqrt[n]{2} \cdot a)^n = c^n.$$

Taking nth root from both sides we get

$$\sqrt[n]{2}a = c$$

or

$$\sqrt[n]{2}a = -c.$$

Now the left sides of these equations are irrational and the right sides of the equations are integers so the only integers a and c that satisfy either of these are $a = c = 0$.

The proof is now complete.

Chapter 2

A Theorem Related to Fermat's Last Theorem

We present a theorem related to Fermat's Last Theorem.

2.1 Introduction

Fermat's Last Theorem states that $a^n + b^n \neq c^n$, $a, b, c \in Z_+, n \in Z, n \geq 3$. We will prove that $a^n + b^n < c^n$, when $a, b \leq c$ and $\sqrt[n]{2} - \frac{1}{c-1} < 1$.

2.2 A Proof

It has to be that $a, b < c$. The left side of the inequation is biggest, when $a, b = c - 1$. Let us show when $a^n + b^n < c^n$: Let $r + 1 = c$. We show when $(r+1)^n > 2r^n$. This is the same as $(r+1)^n - 2r^n > 0$.

$$
\begin{gathered}
(r+1)^n - 2r^n > 0 \\
\Leftrightarrow (r+1)^n - (\sqrt[n]{2}r)^n > 0 \\
\Leftrightarrow r + 1 - \sqrt[n]{2}r > 0 \\
\Leftrightarrow r(1 - \sqrt[n]{2}) + 1 > 0 \\
\Leftrightarrow r(1 - \sqrt[n]{2} + \frac{1}{r}) > 0 \\
\Leftrightarrow 1 - \sqrt[n]{2} + \frac{1}{r} > 0 \\
\Leftrightarrow (\sqrt[n]{2} - \frac{1}{r}) < 1.
\end{gathered}
$$

This concludes our proof. This equation holds when n is large and r is small.

Chapter 3

Infinity Is Not an Integer

3.1 Introduction

Fermat's Last Theorem states that $a^n + b^n \neq c^n$, $a, b, c \in Z_+, n \in Z, n \geq 3$. We will prove that infinity is not an integer using this theorem.

3.2 A Proof

Let us write the Fermat's Last Theorem in the form $c = \sqrt[n]{a^n + b^n}$. We can then write also $\lim_{n\to\infty} \sqrt[n]{a^n + b^n} = \max(a, b)$, that is, the infinity norm. Let us assume that infinity would be an integer. Then infinity were a number and we can write $\lim_{n\to\infty} \sqrt[n]{a^n + b^n} = \sqrt[\infty]{a^\infty + b^\infty} = \max(a, b)$. Since a and b are integers, c would be an integer. But this is a contradiction to Fermat's Last Theorem. So infinity is not an integer.

Chapter 4

References

[1] Wiles, Andrew (1995). "Modular elliptic curves and Fermat's Last Theorem", Annals of Mathematics. 141 (3): 448.