Collected Papers 2023 - A Special Issue on Electric, Magnetic, and Gravitational Fields and Astronomy

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Preface

This article collection has four articles dealing with electric, magnetic, and gravitational fields and three articles dealing with astronomy. The connections of the papers to each other can be seen in Figure 1. To cite an article in this collection, you may do it the following way:

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Joensuu, Finland 1st January, 2024

Mikko I. Malinen

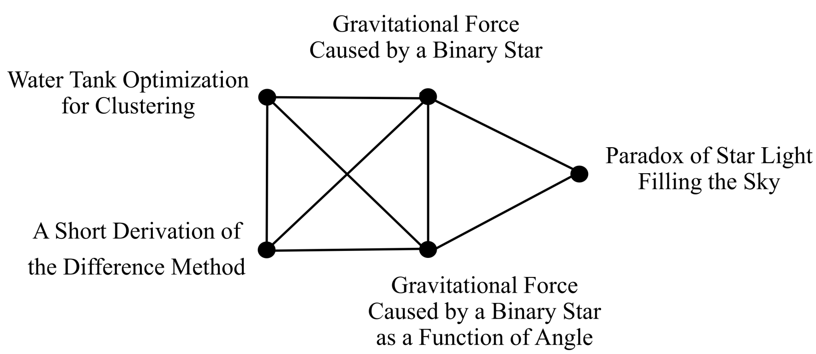

Figure 1: Connections of the Papers to Each Other.

Contents

Chapter 1

A Short Derivation Of the Difference Method For the Laplace Equation

1.1 Introduction

Laplace equation holds in electrostatic and magnetostatic fields [1]. We give a short derivation of the difference method which is also called Liebmann's method in [2], for approximately solving Laplace equation in 2-d, 3-d and higher dimensions.

1.2 Two-Dimensional Case

Laplace equation in two dimensions is

$$\nabla^2 u = u_{xx} + u_{yy} \tag{1.1}$$

We derive the difference method, where field in point i, j equals the mean field of its neighbors. The point and its neighboring points are shown in Figure 1.1. The derivative between u_{ij} and c can be approximated by

$$\frac{c - u_{ij}}{h} \tag{1.2}$$

and the derivative between a and u_{ij} can be approximated by

$$\frac{u_{ij} - a}{h} \tag{1.3}$$

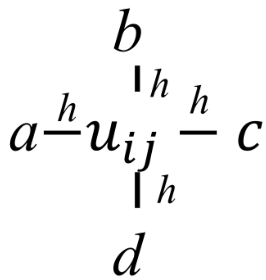

Figure 1.1: A 2-d point u_{ij} and its neighbors a, b, c, d.

and using these the second x-derivative in point u_{ij} can be approximated by

$$\begin{aligned} u_{xx} &\approx \frac{\frac{c-u_{ij}}{h} - \frac{u_{ij}-a}{h}}{h} \\ &= \frac{c-2u_{ij}+a}{h^2} \end{aligned} \tag{1.4}$$

Correspondingly, in y-dimension the second y-derivative is

$$\begin{aligned} u_{yy} &\approx \frac{\frac{b-u_{ij}}{h} - \frac{u_{ij}-d}{h}}{h} \\ &= \frac{b-2u_{ij}+d}{h^2} \end{aligned} \tag{1.5}$$

From Equations (1.1),(1.4) and (1.5) we get

$$\frac{c-2u_{ij}+a+b-2u_{ij}+d}{h^2} = 0 \tag{1.6}$$

$$\Rightarrow a+b+c+d-4u_{ij} = 0 \tag{1.7}$$

This approximative Equation 1.7 is accurate, when $h \to 0$.

1.3 Three-Dimensional Case

3-dimensional Laplace equation is

$$\nabla^2 u = u_{xx} + u_{yy} + u_{zz} = 0 \tag{1.8}$$

Correspondinly, as in two-dimensional case we get with the help of Figure 1.2 in three dimensions

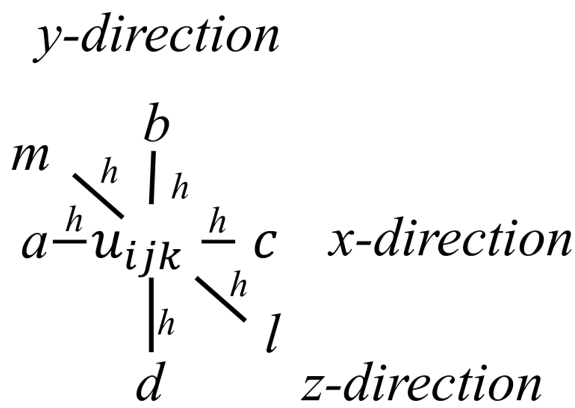

Figure 1.2: A 3-d point u_{ijk} and its neighbors a, b, c, d, l, m.

$$u_{xx} \approx \frac{\frac{c-u_{ijk}}{h} - \frac{u_{ijk}-a}{h}}{h} = \frac{c - 2u_{ijk} + a}{h^2} \tag{1.9}$$

In y-dimension the second y-derivative is

$$u_{yy} \approx \frac{\frac{b-u_{ijk}}{h} - \frac{u_{ijk}-d}{h}}{h} = \frac{b - 2u_{ijk} + d}{h^2} \tag{1.10}$$

and in z-dimension

$$u_{zz} \approx \frac{\frac{l-u_{ijk}}{h} - \frac{u_{ijk}-m}{h}}{h} = \frac{l - 2u_{ijk} + m}{h^2} \tag{1.11}$$

By combining Equations (1.8), (1.9), (1.10) and (1.11) we get

$$\frac{a+b+c+d+l+m-6u_{ijk}}{h^2} = 0 \tag{1.12}$$

$$\Rightarrow a+b+c+d+l+m-6u_{ijk} = 0 \tag{1.13}$$

The Equation 1.13 is accurate, when $h \to 0$.

1.4 Higher dimensionalities

This method scales to any dimensionality, just calculate the field in point u as the mean of its neighboring points.

1.5 Summary

Difference method is extended to 3-d and higher dimensionalities in this article, extending the material in [2].

References

[1] D. K. Cheng, Field and Wave Electromagnetics, Second Edition, Addison-Wesley, 1989

[2] E. Kreyszig, Advanced Engineering Mathematics, Sixth Edition, John Wiley & Sons, 1988

Chapter 2

Gravitational Force Caused By A Binary Star Is Not Constant

The gravitational force caused by two stars that circle around each other to an observer is not constant. The force is maximal when the observer and the stars are on the same line. We showed in [1] that when the stars are of unequal mass, the gravitational force is not constant. In this article we show that when the stars are of equal mass, the gravitational force is not constand either.

2.1 Theory

Consider the case 1 where the stars and the observer are on the same line (Figure 2.1) and the case 2 where the stars are at the same distance from the observer (Figure 2.2). Whe denote the mass of a star by M, and the mass of the observer by m. In Figures 2.1 and 2.2 the distances are shown and calculated. Gravitational force can bre calculated by the formula

$$F = k \cdot \frac{m_1 \cdot m_2}{d^2}, \tag{2.1}$$

where m_1 and m_2 are the masses of the objects, d is the distance between the objects, and k is the gravitational constant. From Equation (2.1) and Figure 2.1 we get the formula for the force affecting the observer in case 1:

$$F_1 = k \cdot \frac{m \cdot M}{(d_2 - \frac{d_1}{2})^2} + k \cdot \frac{m \cdot M}{(d_2 + \frac{d_1}{2})^2} \tag{2.2}$$

$$= k \cdot \frac{m \cdot M}{d_2^2 - 2d_2 \cdot \frac{d_1}{2} + (\frac{d_1}{2})^2} + k \cdot \frac{m \cdot M}{(d_2)^2 + 2d_2 \cdot \frac{d_1}{2} + (\frac{d_1}{2})^2}. \tag{2.3}$$

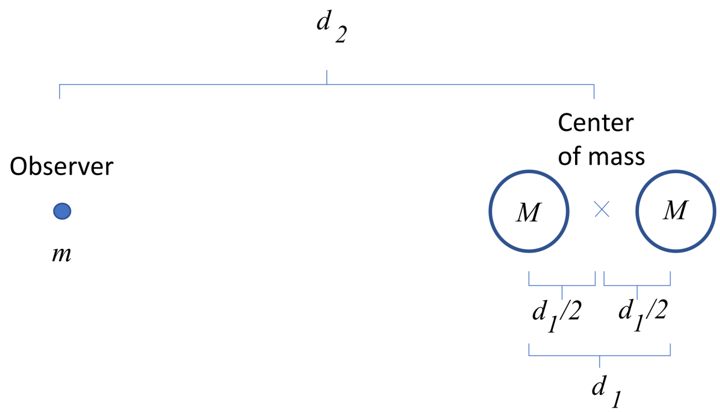

Figure 2.1: Case 1. The stars are on the same line as the observer.

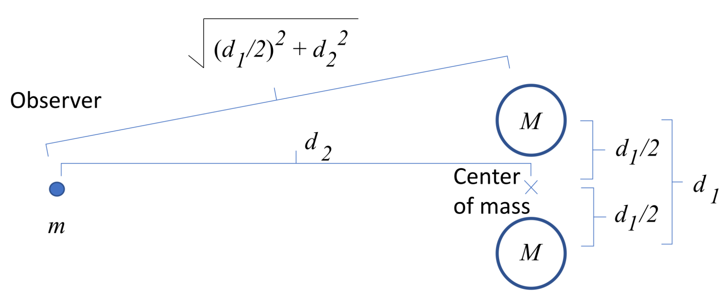

Figure 2.2: Case 1. The stars are at the same distance from the observer. Pythagorean theorem is used.

From Equation (2.1) and Figure 2.2 we get the formula for the force affecting the observer in case 2:

$$F_2 = k \cdot \frac{m \cdot M}{(\frac{d_1}{2})^2 + d_2^2} + k \cdot \frac{m \cdot M}{(\frac{d_1}{2})^2 + d_2^2} \tag{2.4}$$

Without loss of generality we can set $m \cdot M = 1$, $(\frac{d_1}{2})^2 + d_2^2 = 2$ and $2d_2 \cdot \frac{d_1}{2} = 1$. Then

$$F_1 = k \cdot \frac{1}{2-1} + k \cdot \frac{1}{2+1} = k \cdot 1\frac{1}{3} \tag{2.5}$$

$$> F_2 = k \cdot \frac{1}{2} + k \cdot \frac{1}{2} = k \cdot 1 \tag{2.6}$$

$$\Rightarrow F_1 > F_2. \tag{2.7}$$

That is, the gravitational force affecting the observer is not constant.

2.2 Discussion

Gravitational force experienced by an observer was shown to be changing in the case of stars with unequal mass [1]. In this paper we showed that it also holds for stars with equal mass.

References

[1] M. I. Malinen, "On Gravitational Waves", in Collected Papers 2017, Mikko I. Malinen, Joensuu, Finland, 2018

Chapter 3

Gravitational Force Caused By A Binary Star As A Function of Angle

The gravitational force caused by two stars that circle around each other to an observer is not constant. The force is maximal when the observer and the stars are on the same line. We showed in [1] that when the stars are of unequal mass, the gravitational force is not constant. In [2] we showed that when the stars are of equal mass, the gravitational force is not constant either. In this article we give a formula for the gravitational force as a function of the angle of the stars.

3.1 Theory

Consider the case where the angle of the stars is $[0, \pi/2]$, see Figure 3.1). We denote the mass of a star by M, and the mass of the observer by m. Gravitational force can be calculated by the formula

$$F = G \cdot \frac{m_1 \cdot m_2}{d^2}, \tag{3.1}$$

where m_1 and m_2 are the masses of the objects, d is the distance between the objects, and G is the gravitational constant. In the following we give the equations in angle range $[0, \pi/2]$, the results can easily be extended to range $[0, 2\pi]$ by symmetry. From Figure 3.1 we get the distance between the observer

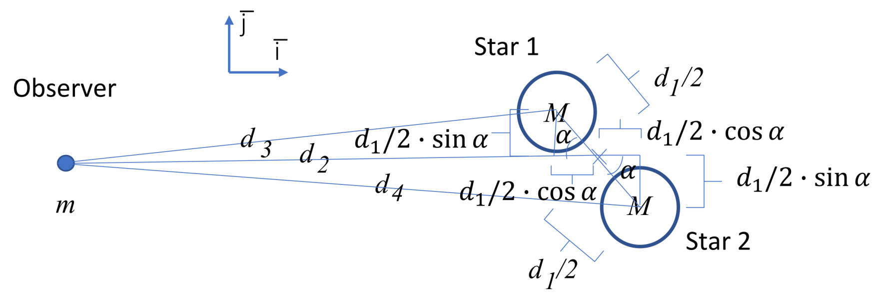

Figure 3.1: The observer, the stars, distances and the angle α.

and the star 1 by using Pythagorean theorem:

$$d_3 = \sqrt{(d_2 - \frac{d_1}{2}\cos(\alpha))^2 + (\frac{d_1}{2}\sin(\alpha))^2} \tag{3.2}$$

$$= \sqrt{d_2^2 + \frac{d_1^2}{4}\cos^2(\alpha) - d_1 d_2 \cos\alpha + \frac{d_1^2}{4}\sin^2(\alpha)} \tag{3.3}$$

$$= \sqrt{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha} \tag{3.4}$$

From Figure 3.1 we get the distance between the observer and the star 2 by using Pythagorean theorem:

$$d_4 = \sqrt{(d_2 + \frac{d_1}{2}\cos\alpha)^2 + (\frac{d_1}{2}\sin\alpha)^2} \tag{3.5}$$

$$= \sqrt{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}\cos^2\alpha + \frac{d_1^2}{4}\sin^2\alpha} \tag{3.6}$$

$$= \sqrt{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}} \tag{3.7}$$

In the following formula we write the resultant force between the observer and the two stars. Horizontal direction is denoted by unit vector $\bar{i}$ and vertical direction by unit vector $\bar{j}$. The formula repeats four times the pattern "direction"·"gravitational force". Two first patterns are the contribution of star 1 and the two last patterns are the contribution of the star 2:

$$\bar{F}_{res} = \frac{\frac{d_1}{2}\sin\alpha}{\sqrt{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha}}\bar{j} \cdot G \cdot \left(\frac{mM}{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha}\right) \tag{3.8}$$

$$+ \frac{d_2 - \frac{d_1}{2}\cos\alpha}{\sqrt{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha}}\bar{i} \cdot G \cdot \left(\frac{mM}{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha}\right) \tag{3.9}$$

$$- \frac{\frac{d_1}{2}\sin\alpha}{\sqrt{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}}}\bar{j} \cdot G \cdot \left(\frac{mM}{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}}\right) \tag{3.10}$$

$$+ \frac{d_2 + \frac{d_1}{2}\cos\alpha}{\sqrt{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}}}\bar{i} \cdot G \cdot \left(\frac{mM}{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}}\right) \tag{3.11}$$

We next calculate the absolute value of the resultant force. It does not include unit vectors $\bar{i}$ or $\bar{j}$, because it is the length of the vector $\bar{F}_{res}$. It is obtained from $\bar{F}_{res}$ by Pythagorean theorem:

$$F_{res} = ((\frac{\frac{d_1}{2}\sin\alpha}{\sqrt{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha}} \cdot G \cdot \frac{mM}{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha} \tag{3.12}$$

$$- \frac{\frac{d1}{2}\sin\alpha}{\sqrt{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}}} \cdot G \cdot \frac{mM}{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}})^2 \tag{3.13}$$

$$+ (\frac{d_2 - \frac{d_1}{2}\cos\alpha}{\sqrt{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha}} \cdot G \cdot \frac{mM}{d_2^2 + \frac{d_1^2}{4} - d_1 d_2 \cos\alpha} \tag{3.14}$$

$$+ \frac{d_2 + \frac{d_1}{2}\cos\alpha}{\sqrt{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}}} \cdot G \cdot \frac{mM}{d_2^2 + d_1 d_2 \cos\alpha + \frac{d_1^2}{4}})^2)^{\frac{1}{2}} \tag{3.15}$$

The resultant angle γ relative to direction $\bar{i}$ can be obtained from $\bar{F}_{res}$ by using arctan()-function:

$$\gamma = \arctan(\frac{\frac{\frac{d_1}{2}\sin\alpha}{\sqrt{d_2^2+\frac{d_1^2}{4}-d_1 d_2\cos\alpha}} \cdot G \cdot \frac{mM}{d_2^2+\frac{d_1^2}{4}-d_1 d_2\cos\alpha}}{\frac{d_2-\frac{d_1}{2}\cos\alpha}{\sqrt{d_2^2+\frac{d_1^2}{4}-d_1 d_2\cos\alpha}} \cdot G \cdot \frac{mM}{d_2^2+\frac{d_1^2}{4}-d_1 d_2\cos\alpha}} \ldots \tag{3.16}$$

$$\ldots \frac{-\frac{\frac{d1}{2}\sin\alpha}{\sqrt{d_2^2+d_1 d_2\cos\alpha+\frac{d_1^2}{4}}} \cdot G \cdot \frac{mM}{d_2^2+d_1 d_2\cos\alpha+\frac{d_1^2}{4}}}{+\frac{d_2+\frac{d_1}{2}\cos\alpha}{\sqrt{d_2^2+d_1 d_2\cos\alpha+\frac{d_1^2}{4}}} \cdot G \cdot \frac{mM}{d_2^2+d_1 d_2\cos\alpha+\frac{d_1^2}{4}}}) \tag{3.17}$$

3.2 Discussion

We have calculated the gravitational force affecting the observer by a binary star where the stars circle around each other. We note that even in this simple case the equations get complicated. One other note is that in the resultant angle γ equation, m and M cancel out if the equation is simplified. This means that the masses of the observer and the stars do not affect the resultant angle γ. We calculated the resultant force in a vector form and in absolute value.

References

[1] M. I. Malinen, "On Gravitational Waves", in Collected Papers 2017, Mikko I. Malinen, Joensuu, Finland, 2018

[2] M. I. Malinen, "Gravitational Force Caused By A Binary Star Is Not Constant", to appear, 2024

Chapter 4

Clustering by Laws of Nature - Abstract

This chapter is an abstract rather than a complete article. We aim at publishing a complete article later. The idea is that the points of a dataset have positive electric charges and the centroids have negative electric charges. Points will be put fixed in the bottom of a water tank and the centroids may move freely on the upper surface of the water. Centroids and points attract each other and centroids repel other centroids, forces are given by the Coulomb's law. We expect that, when charges and water height are set correctly, this gives a moderate clustering result.

Chapter 5

Paradox of Star Light Filling the Sky

We calculate what proportion the sky should be filled with stars. On the other hand, the further we look, the more volume of space is included in our view, but on the other hand a mean sized star occupies smaller proportion of space when we look further. We assume here that all stars are mean sized and that in a certain volume of space there is a constant number of stars.

5.1 Calculation

We look from the surface of Earth, that is about 6400 km from the center point of Earth. Volume change with respect to radius change is

$$\frac{dV}{dr} = 4\pi r^2 \tag{5.1}$$

Proportion of stars occupying sky within radius r is

$$A = \frac{a}{r}, \tag{5.2}$$

where a is the proportion at radius 1. From (5.2) we get

$$\frac{dA}{dr} = -\frac{a}{r^2} \tag{5.3}$$

From (5.1) and (5.3) we get

$$\frac{dV}{dr} + \frac{dA}{dr} = 4\pi r^2 - \frac{a}{r^2} \tag{5.4}$$

Integrating the right side of (5.4) from the surface of Earth to radius R we get

$$\int_{6400}^{R} 4\pi r^2 - \frac{a}{r^2} dr = /_{6400}^{R} \frac{4}{3}\pi r^3 - \frac{a}{r} = \frac{4}{3}\pi R^3 - \frac{a}{R} - \frac{4}{3}\pi \cdot 6400^2 + \frac{a}{6400} \tag{5.5}$$

From (5.5) we see that the further we look and let R increase the more the sky is filled with stars and eventually the sky should be totally filled with stars. This is contradictory to what we see on the sky.

5.2 Discussion

If you know an explanation for the sky not filled with light, I am interested to hear, my email address is on the title page.