Collected Papers First Half of 2024 - A Special Issue on Special Relativity and Cosmology

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Preface

This article collection has four articles dealing with Special Relativity and two articles dealing with cosmology. The topics vary from amendments to Einstein's articles to problems I have found from Special Relativity. To cite an article in this collection, you may do it the following way:

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Chapter 1

Derivation of the Special Theory of Relativity's Time Dilation

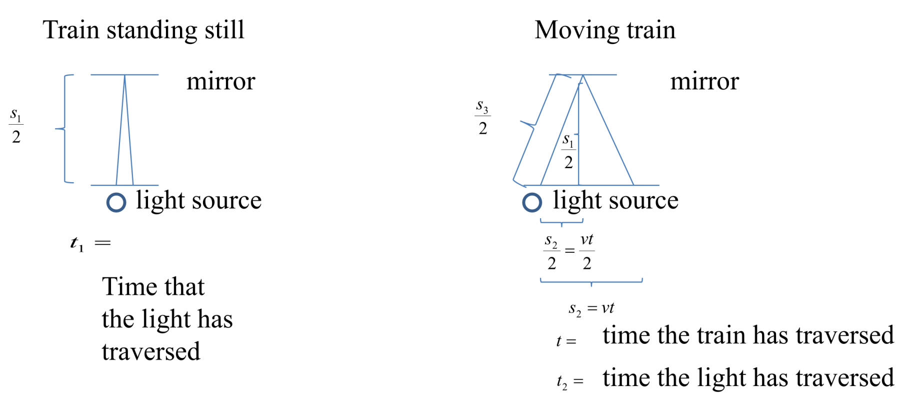

Figure 1. The light's route from the source to the observer.

1.1 Introduction

In this article we derive the expression for the special theory of relativity's time dilation

$$\frac{1}{\sqrt{1 - (\frac{v}{c})^2}}.$$

This article is a translation from an article written in Finnish [1].

1.2 Derivation

Let's assume that a light source is put on the floor of a train. It's light goes via a mirror at the ceiling, back to an observer on the floor (Figure 1.). When the train is moving, the light has to traverse a longer route (Figure 1 right). We calculate how much the length of the route changes relative to the case of Figure 1 left, that is, by the markings of Figure 1, the relation s_3/s_1. From the settings of Figure 1 and from the Pythagorean theorem we get

$$(\frac{s_3}{2})^2 = (\frac{s_1}{2})^2 + (\frac{vt}{2})^2 = (\frac{s_1}{2})^2 + (\frac{vs_3}{2c})^2,$$

because $t_1 = \frac{s_1}{c}$ and $t_2 = \frac{s_3}{c}$ and $t = \frac{s_3}{c}$ and $t_1 \neq t_2 = t$.
So we have

$$(\frac{s_3}{2})^2 = (\frac{s_1}{2})^2 + (\frac{vs_3}{2c})^2,$$

from which we get

$$\begin{aligned}
\frac{s_3^2}{4} - \frac{v^2 s_3^2}{4c^2} &= (\frac{s_1}{2})^2 \\
\Rightarrow \frac{c^2 s_3^2}{4c^2} - \frac{v^2 s_3^2}{4c^2} &= (\frac{s_1}{2})^2 \\
\Rightarrow \frac{s_3^2(c^2 - v^2)}{4c^2} &= \frac{s_1^2}{4} \\
\Rightarrow (\frac{s_3}{s_1})^2 &= \frac{4c^2}{4(c^2 - v^2)} \\
\Rightarrow (\frac{s_3}{s_1})^2 &= \frac{c^2}{c^2 - v^2} \\
\Rightarrow (\frac{s_1}{s_3})^2 &= \frac{c^2 - v^2}{c^2} \\
&= 1 - (\frac{v}{c})^2 \\
\Rightarrow \frac{s_3}{s_1} &= \frac{1}{\sqrt{1 - (\frac{v}{c})^2}},
\end{aligned}$$

that is the time dilatation.

References

[1] Mikko I. Malinen, "Suhteellisuusteorian aikadilataation johto", in Mikko I. Malinen, Collected Papers 2017, Joensuu, Finland, 2018

Chapter 2

Mass is Energy - Some Amendments to Einstein's Paper

2.1 Introduction

We list here the additions which should be placed between Formulas

$$K_0 - K_1 = L\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\} \tag{2.1}$$

and

$$K_0 - K_1 = \frac{1}{2}\frac{L}{c^2}v^2 \tag{2.2}$$

on Page 3 of Einstein's original paper [1] (Figure 2.3).

2.2 Additions

Let's write

$$x = \frac{v}{c} \tag{2.3}$$

and

$$f(a) = L(\frac{1}{\sqrt{1 - a^2 - 1}}). \tag{2.4}$$

DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY-CONTENT?

By A. EINSTEIN

THE results of the previous investigation lead to a very interesting conclusion, which is here to be deduced.

I based that investigation on the Maxwell-Hertz equations for empty space, together with the Maxwellian expression for the electromagnetic energy of space, and in addition the principle that:—

The laws by which the states of physical systems alter are independent of the alternative, to which of two systems of co-ordinates, in uniform motion of parallel translation relatively to each other, these alterations of state are referred (principle of relativity).

With these principles * as my basis I deduced *inter alia* the following result (§ 8):—

Let a system of plane waves of light, referred to the system of co-ordinates (x, y, z), possess the energy l; let the direction of the ray (the wave-normal) make an angle ϕ with the axis of x of the system. If we introduce a new system of co-ordinates (ξ, η, ζ) moving in uniform parallel translation with respect to the system (x, y, z), and having its origin of co-ordinates in motion along the axis of x with the velocity v, then this quantity of light—measured in the system (ξ, η, ζ) —possesses the energy

$$l^* = l\frac{1 - \frac{v}{c}\cos\phi}{\sqrt{1 - v^2/c^2}}$$

* The principle of the constancy of the velocity of light is of course contained in Maxwell's equations.

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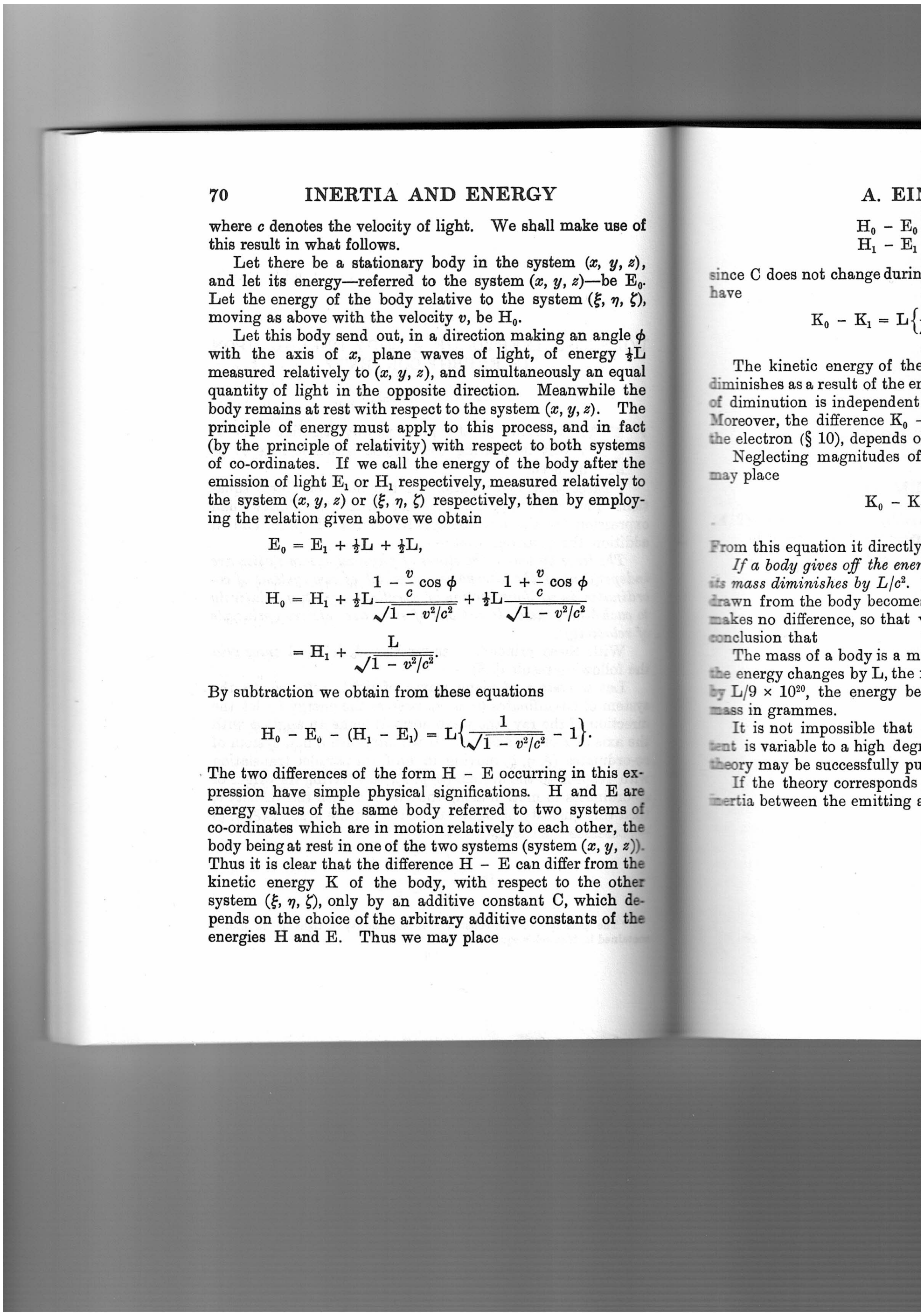
Figure 2.1: Page 1



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where c denotes the velocity of light. We shall make use of this result in what follows.

Let there be a stationary body in the system (x, y, z), and let its energy—referred to the system (x, y, z)—be E_0. Let the energy of the body relative to the system (ξ, η, ζ), moving as above with the velocity v, be H_0.

Let this body send out, in a direction making an angle ϕ with the axis of x, plane waves of light, of energy $\frac{1}{2}L$ measured relatively to (x, y, z), and simultaneously an equal quantity of light in the opposite direction. Meanwhile the body remains at rest with respect to the system (x, y, z). The principle of energy must apply to this process, and in fact (by the principle of relativity) with respect to both systems of co-ordinates. If we call the energy of the body after the emission of light E_1 or H_1 respectively, measured relatively to the system (x, y, z) or (ξ, η, ζ) respectively, then by employing the relation given above we obtain

$$E_0 = E_1 + \tfrac{1}{2}L + \tfrac{1}{2}L,$$

$$H_0 = H_1 + \tfrac{1}{2}L\frac{1 - \frac{v}{c}\cos\phi}{\sqrt{1 - v^2/c^2}} + \tfrac{1}{2}L\frac{1 + \frac{v}{c}\cos\phi}{\sqrt{1 - v^2/c^2}}$$

$$= H_1 + \frac{L}{\sqrt{1 - v^2/c^2}}.$$

By subtraction we obtain from these equations

$$H_0 - E_0 - (H_1 - E_1) = L\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

The two differences of the form $H - E$ occurring in this expression have simple physical significations. H and E are energy values of the same body referred to two systems of co-ordinates which are in motion relatively to each other, the body being at rest in one of the two systems (system (x, y, z)). Thus it is clear that the difference $H - E$ can differ from the kinetic energy K of the body, with respect to the other system (ξ, η, ζ), only by an additive constant C, which depends on the choice of the arbitrary additive constants of the energies H and E. Thus we may place

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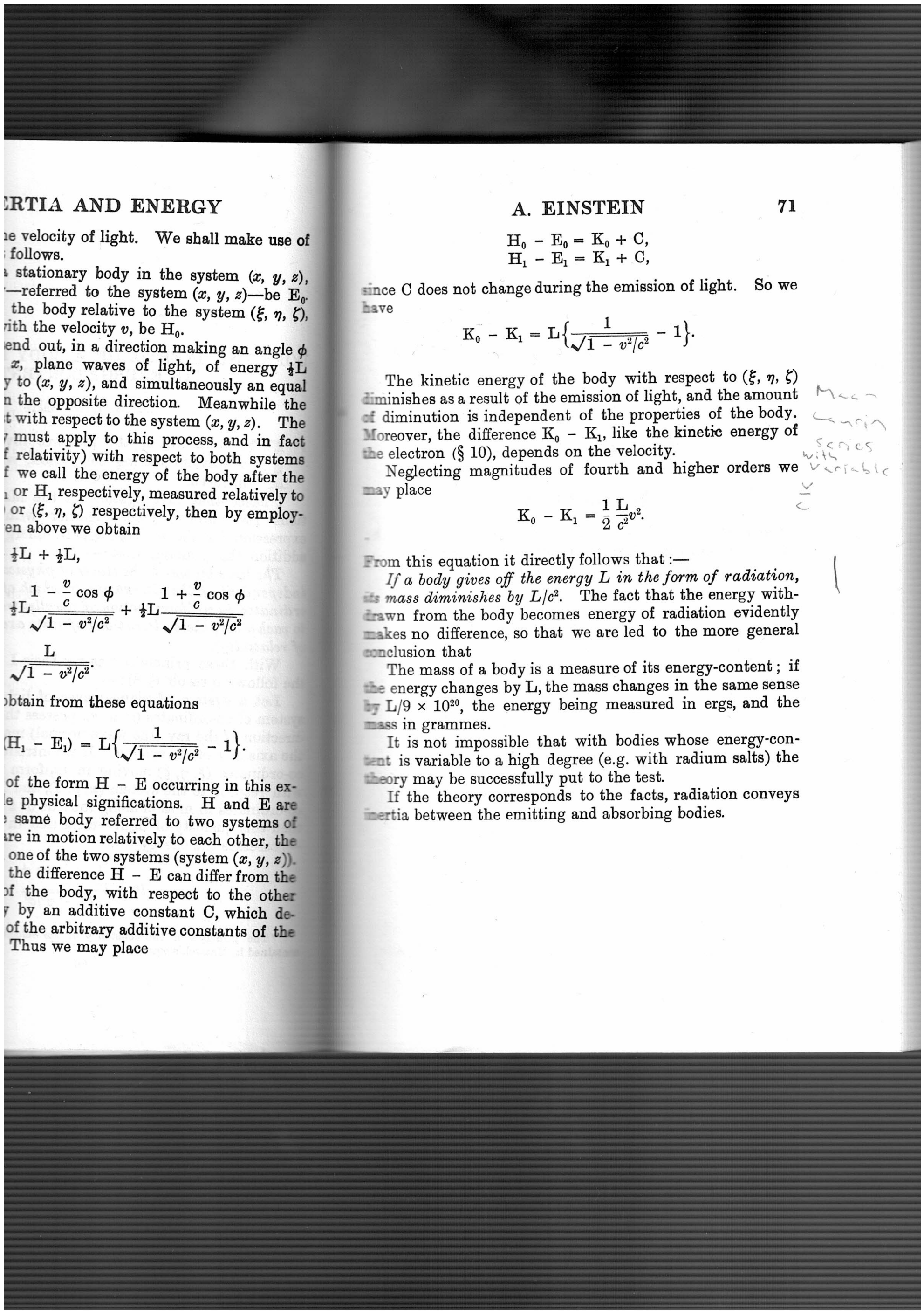

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$$\ldots \tfrac{1}{2}L + \tfrac{1}{2}L,$$

$$\ldots \tfrac{1}{2}L\frac{1 - \frac{v}{c}\cos\phi}{\sqrt{1 - v^2/c^2}} + \tfrac{1}{2}L\frac{1 + \frac{v}{c}\cos\phi}{\sqrt{1 - v^2/c^2}}$$

$$\ldots \frac{L}{\sqrt{1 - v^2/c^2}}.$$

btain from these equations

$$\ldots (H_1 - E_1) = L\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

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of the arbitrary additive constants of the
Thus we may place

A. EINSTEIN 71

$$H_0 - E_0 = K_0 + C,$$
$$H_1 - E_1 = K_1 + C,$$

since C does not change during the emission of light. So we have

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

The kinetic energy of the body with respect to (ξ, η, ζ) diminishes as a result of the emission of light, and the amount of diminution is independent of the properties of the body. Moreover, the difference $K_0 - K_1$, like the kinetic energy of the electron (§ 10), depends on the velocity.

Neglecting magnitudes of fourth and higher orders we may place

$$K_0 - K_1 = \frac{1}{2}\frac{L}{c^2}v^2.$$

From this equation it directly follows that:—

If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2. The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that

The mass of a body is a measure of its energy-content; if the energy changes by L, the mass changes in the same sense by $L/9 \times 10^{20}$, the energy being measured in ergs, and the mass in grammes.

It is not impossible that with bodies whose energy-content is variable to a high degree (e.g. with radium salts) the theory may be successfully put to the test.

If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.

Figure 2.3: Page 3

Then, using chain rule and the formula for the derivative of a quotient,

$$f'(a) = \frac{d}{da} L(\frac{1}{\sqrt{1-a^2}} - 1) \tag{2.5}$$
$$= L(\frac{-1 \cdot \frac{1}{2}(1-a^2)^{-1/2} \cdot -2a}{1-a^2}) \tag{2.6}$$
$$= L(\frac{a}{(1-a^2)^{3/2}}) \tag{2.7}$$

and

$$f''(a) = \frac{d}{da} L(\frac{a}{(1-a^2)^{3/2}}) \tag{2.8}$$
$$= L(\frac{(1-a^2)^{3/2} - a \cdot \frac{3}{2}(1-a^2)^{1/2} \cdot -2a}{(1-a^2)^3}) \tag{2.9}$$
$$= L(\frac{(1-a^2) + \frac{3}{2} \cdot 2a^2}{(1-a^2)^{2,5}}) \tag{2.10}$$
$$= L\frac{(1-a^2) + 3a^2}{(1-a^2)^{2,5}}) \tag{2.11}$$
$$= L(\frac{1+2a^2}{(1-a^2)^{2,5}}) \tag{2.12}$$

and

$$f'''(a) = \frac{d}{da} L(\frac{1+2a^2}{(1-a^2)^{2,5}}) \tag{2.13}$$
$$= L(\frac{4a \cdot (1-a^2)^{2,5} - (1+2a^2) \cdot \frac{5}{2}(1-a^2)^{1,5} \cdot -2a}{(1-a^2)^5}) \tag{2.14}$$
$$= L(\frac{4a \cdot (1-a^2) - (1+2a^2) \cdot \frac{5}{2} \cdot -2a}{(1-a^2)^{3,5}}. \tag{2.15}$$

Maclaurin polynomial, that is, Taylor polynomial at point $a = 0$, is

$$P_3(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 \tag{2.16}$$
$$= 0 + 0 + \frac{L}{2}x^2 + 0 \tag{2.17}$$
$$= \frac{L}{2}x^2 \tag{2.18}$$
$$= \frac{1}{2}\frac{L}{c^2}v^2. \tag{2.19}$$

The last line is the result written in Einstein's paper.

References

[1] H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, "The Principle of

Relativity", Dover Publications, Reprint of 1923 translation

Chapter 3

An Alternative Way to Derive the Equivalence of Mass and Energy

3.1 Introduction

In this article we derive the equivalence of mass and energy, as in [1], p.279.

3.2 Derivation

In Einstein's special theory of relativity, the momentum of a particle having rest mass m_0 moving at speed v is given by

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{3.1}$$

where c is the speed of light. This is consistent with the particle's having an apparent mass of $m = m_0/\sqrt{1 - (v^2/c^2)}$ if we use the Newtonian interpretation of momentum as mass × velocity. Note that $m \to m_0$ as $v \to 0+$ and $m \to \infty$ as $v \to c-$.
Using the linearization of $(1-t)^{-1/2}$ about $t = 0$, namely $L(t) = 1 + (t/2)$, which is a good approximation to $(1-t)^{-1/2}$ for small values of $|t|$, we conclude, taking $t = v^2/c^2$, that if v is small compared to c, then

$$m \approx m_0(1 + \frac{v^2}{2c^2}) = m_0 + \frac{1}{c^2}(\frac{1}{2}m_0 v^2). \tag{3.2}$$

In going from speed 0 to speed v, the apparent mass of the particle has increased by

$$\Delta m = m - m_0 \approx \frac{1}{c^2}\Delta KE, \tag{3.3}$$

where

$$\Delta KE = \frac{1}{2} m_0 (v^2 - 0^2) = \frac{1}{2} m_0 v^2 \tag{3.4}$$

is the change in the (classical) kinetic energy of the particle. Thus the increase in kinetic energy is about c^2 times the increase in mass.

References

[1] Robert A. Adams, "Calculus: A Complete Course", fourth edition, Addison Wesley, 1999

Chapter 4

Time Dilation in Special Relativity Is Not Transitive

4.1 Introduction

I report that the time dilation in special relativity is not transitive.

4.2 Transitivity

Let elapsed time in a point in space be t_0. A spaceship is sent from that point in space with velocity v to a certain direction. Elapsed time on this first spaceship is

$$\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{4.1}$$

A second spaceship is sent from that point with velocity $2v$ to the same direction as the first spaceship. Elapsed time on the second spaceship expressed in terms of the time of the first spaceship is

$$\frac{t_1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t_0}{1-\frac{v^2}{c^2}}. \tag{4.2}$$

Elapsed time on the second spaceship expressed in terms of the time on the initial point t_0 is

$$\frac{t_0}{\sqrt{1-\frac{4v^2}{c^2}}}. \tag{4.3}$$

Now Expression (4.2) $\neq$ Expression (4.3). This shows that the time dilation in special relativity is not transitive. This puts the validity of special relativity into question.

Chapter 5

There Is No Space Without Time

5.1 Introduction

This article is in the field of cosmology. We show that if there is space, there is always time.

5.2 Derivation

A law of physics says that

$$v = \frac{dx}{dt}, \tag{5.1}$$

that is, velocity is defined to be change of location divided by change of time. If there is no time t, that is, time is undefined, then also velocity v is undefined. That means, there is no velocity, and further, there is no space.

Chapter 6

There Is No Time Without Space

6.1 Introduction

This article is in the field of cosmology. We show that if there is time, there is always space.

6.2 Derivation

A law of physics says that

$$s = vt, \tag{6.1}$$

that is, change of location is defined to be velocity times time. This law of physics can be written in the form

$$\frac{s}{v} = t \tag{6.2}$$

In Equation (6.2), if there is no space (s and v are not defined), then also time t is undefined. That means, in this case there is no time.