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COORDINATE QUANTIZATION IN VECTOR MAP COMPRESSION

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ABSTRACT

We consider the quantization problem of lossy vector map compression. The compression is performed by scalar quantization. The scalar quantization is processed in optimal way: we use the Dynamic programming quantization algorithm instead of using uniform or locally optimal Max-Lloyd algorithms. This approach allows us to increase the efficiency of lossy compression for product scalar quantization in rate-distortion sense. We also consider the problem of increasing speed of convergence of existing vector quantization algorithm: randomized local search. The proposed method of using the optimal product scalar codebook as initial codebook increases the speed of convergence for RLS algorithm.

KEYWORDS: vector map compression, polar quantization, Cartesian quantization, vector quantization, dynamic programming

1. Introduction

The goal of vector map compression is to find a compact representation of map, with some limited sacrifice of spatial accuracy. Lossy compression schemes are acceptable as long as the systems can accurately infer the places of interests (e.g. a video store, train route, road name and city block) with a location device such as a GPS. Cartographers routinely use generalization to highlight key features in a map by introducing bounded distortions, e.g. errors in the location of spatial objects.

We consider lossy compression of coordinates by quantization. The compression procedure consists of three main steps (Fig. 1): transformation of input vector data, quantization, and entropy coding of quantized data.

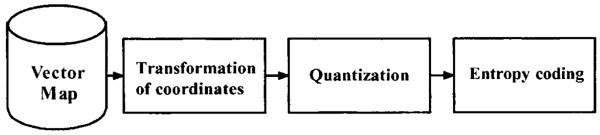

Fig. 1. Principal scheme of the compression algorithm

We considered two models of map data compression. The first model supposes to have an independent codebook for each map. This codebook need to be stored in the compressed file. The given method demands to solve combined problem of minimizing distortion of quantization with minimizing the codebook storage place requirements. The second model consists of building of a single codebook for a map collection and using the given codebook for all maps. This codebook is generated according to all maps from the collection, or by some part of them. In this case, we do not need to store the codebook in the compressed file because it will be included in the decoder.

For solution of the first problem (compression with storage of the codebook) were used the product quantizers. We have considered two types of product quantizer: with Cartesian and polar coordinates. The advantage of them that optimal solution for the quantizers can be guaranteed by a relatively fast algorithm in $O(MN)$ time, and that the storage requirement for codebook is sub-linear $O(\sqrt{M})$, where N is the number of quantized elements, and M is the codebook size.

For the solution of the second type problem (compression without storage of the codebook) we have used vector quantization. As the optimal vector

Fig 2. The test sets #1 and #2: a) The shoreline of Australia; 2903 points, b) The shoreline of Britain; 10909 points.

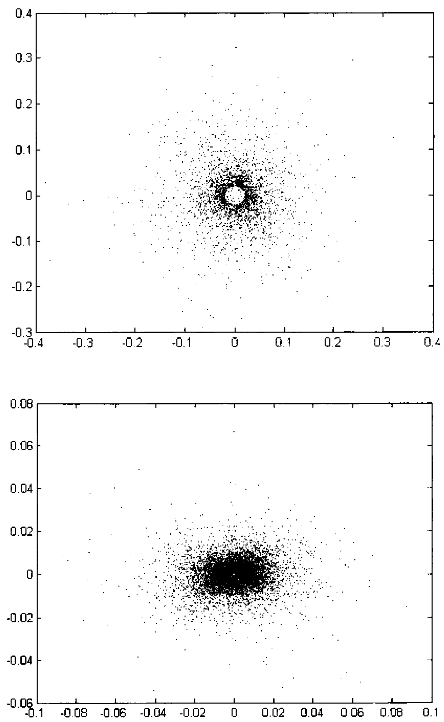

Fig 3. The set of relative coordinates for test set #1 (top) and test set #2(down)

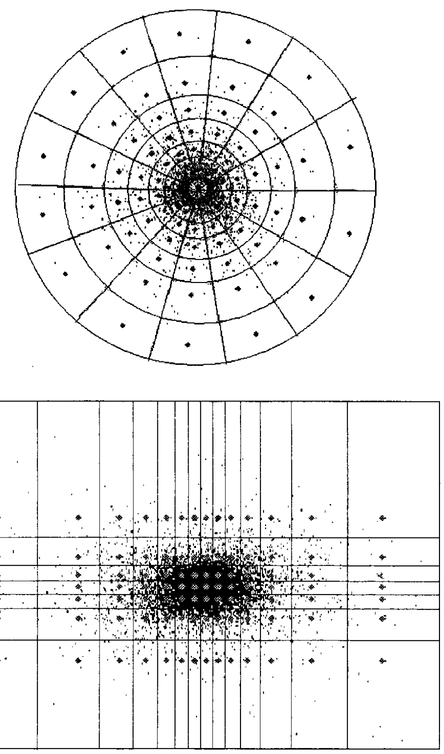

Fig. 4. Strictly polar quantizer (top) and 2D product Cartesian quantizer (down). Centroids are marked as dot points.

quantization problem is NP-hard, suboptimal solution was found by the *Randomized Local Search* (RLS) [1]. The RLS algorithm is a locally optimal iterative algorithm. Its performance depends on the number of iterations, and we propose to use optimal product scalar quantization codebook as an initial codebook for RLS to increase the convergence of RLS and, consequently, decrease the number of iterations needed.

2. Coordinates transformation

The encoding is processed in DPCM manner: we encode the relative coordinates Δx_i and Δy_i:

$$\Delta x_i = x_i - x_{i-1}$$
$$\Delta y_i = y_i - y_{i-1}$$

The resulting 2D set of prediction errors (Δx_i, Δy_i) forms the input data set for quantization. See Fig. 3 for example.

3. Two-dimensional quantization

According to the compression problem, when it is needed to store the codebook in the compressed file, we use the *product quantizer.*

Product quantizer for 2D-space is a quantizer with following structure: $Q : R^2 \to C_1 \times C_2$. This quantizer is the mapping from 2D-space to the product set $C_1 \times C_2$. It means that we are quantizing all vector components as a set of independent variables, and the quantizer Q can be considered as a vector-function $Q = (Q_1, Q_2)$, where Q_i is mapping from one-dimensional space to the scalar codebook.

The principle choice of using the product quantizer for this problem is made because of its low memory requirements. To describe 2D product quantizer's codebooks C_1 and C_2 we need memory about: $L_1 = F \cdot (M_1 + M_2) + 2 \cdot I$, where F is the number of bytes, needed to represent float type number, I is the number of bytes needed to represent an integer number, M_1 and M_2 are the sizes of codebooks for the dimensions. In general, this value is always smaller than the number of bytes, needed to describe each element of the total codebook $C_1 \times C_2$: $L_2 = 2 \cdot F \cdot (M_1 \cdot M_2) + 1 \cdot I$.

We consider *product Cartesian quantizer* (CQ) and *strictly polar quantizer* (PQ). Voran and Scharf [2] used Max-Lloyd algorithm to construct strictly polar quantizer, but the method cannot guarantee optimality of the solution. In [3, 4, 5, 6, 7] the problem of optimal polar quantization was solved analytically or numerically under assumption of uniform phase distribution. The approach provides asymptotically optimal solution for signal with known probability density function for special cases only (2-dimensional Gaussian signal).

3.1 Product Cartesian quantization

Product scalar Cartesian quantizer for the data is described by rectangular grid of centroids $\{\bar{x}_j\}$ and $\{\bar{y}_k\}$ for the coordinates x and y, respectively (see Fig.4); the total number of cells is $M=M_xM_y$. Quantization cell $C_{j,k}$ of the product Cartesian quantizer is defined as intersection of cells X_j and Y_k for dimensions x and y: $C_{j,k}=X_j\cap Y_k$. Consider the mean square error $E(M)$ of the product scalar quantization of the 2-dimensional variable $\xi=(x,y)$:

$$E=\sum_{j=1}^{M_x}\sum_{k=1}^{M_y}\sum_{\xi_n\in C_{j,k}} p_{xy}(x_n,y_n)[(x_n-\bar{x}_j)^2+(y_n-\bar{y}_k)^2].$$

The error $E(M)$ can be represented as summa of quantization errors for two independent scalar quantizers:

$$E=\sum_{j=1}^{M_x}\sum_{x_n\in X_j} p_x(x_n)(x_n-\bar{x}_j)^2+\sum_{k=1}^{M_y}\sum_{y_n\in Y_k} p_y(y_n)(y_n-\bar{y}_k)^2.$$

Problem of optimal 2-dimensional Cartesian quantization for a given total number of clusters M can be formulated as an optimization problem:

$$E(M)=\min_{M_x\cdot M_y}\{\min_{\{\bar{x}_j\}}\{\sum_{j=1}^{M_x}\sum_{x_n\in X_j} p_x(x_n)(x_n-\bar{x}_j)^2+$$

$$+\min_{\{\bar{y}_k\}}\{\sum_{k=1}^{M_y}\sum_{y_n\in Y_k} p_y(y_n)(y_n-\bar{y}_k)^2\}\}.$$

subject to: $M_xM_y\leq M_0$

Introducing rate-distortion functions $G_x(M_x)$ and $G_y(M_y)$ as quantization errors for optimal scalar quantizers on x and y we can formulate the optimization problem as follows:

$$E(M)=\min_{M_x\cdot M_y}\{G_x(M_x)+G_y(M_y)\}$$

subject to: $M_xM_y\leq M$.

Minimum of the cost function E has to be found by optimal construction of product scalar quantizer for the Cartesian coordinates, and optimal by choice of cell numbers M_x and M_y for dimensions x and y.

3.2 Strictly polar quantization

Now let us consider problem of 2-dimensional vector data quantization, represented in polar form in polar form $\xi_n=r_ne^{i\varphi_n}$ under assumption that radius r and phase φ are independent: $p(r,\varphi)=p_rp_\varphi$. The strictly polar quantizer is described by two sets of centroids: centroids $\{\bar{r}_j\}$ for radius r, and centroids $\{\bar{\varphi}_k\}$ for phase φ (see Fig.4). The total number of cells is $M=M_rM_\varphi$. Quantization cell $C_{j,k}$ is defined as intersection of ring R_j and segment Ψ_k. Consider the mean square error E of the polar quantization of the variable $\xi_n=r_ne^{i\varphi_n}$:

$$E=\sum_{j=1}^{M_r}\sum_{k=1}^{M_\varphi}\sum_{\xi_n\in C_{j,k}} p_{r,\varphi}(r_n,\varphi_n)\,|\,r_ne^{i\varphi_n}-\bar{r}_je^{i\bar{\varphi}_k}|^2.$$

or

$$E=\sum_{j=1}^{M_r}\sum_{r_n\in R_j} p_r(r_n)(r_n-\bar{r}_j)^2+2\cdot\sum_{k=1}^{M_\varphi}\sum_{\varphi_n\in\Psi_k} p_\varphi(\varphi_n)r_n\bar{r}_j(1-\cos(\varphi_n-\bar{\varphi}_k))$$

Minimization of the cost function E in general form is difficult and time consuming time. Let us approximate the expression for error E as follows:

$$E=\sum_{j=1}^{M_r}\sum_{r_n\in R_j} p_r(r_n)(r_n-\bar{r}_j)^2+\sum_{k=1}^{M_\varphi}\sum_{\varphi_n\in\Psi_k} p_\varphi(\varphi_n)r_n^2(\varphi_n-\bar{\varphi}_k)^2.$$

for the large M_r and M_φ, when $|r_n-\bar{r}_j|<<\bar{r}_j$, and $|\varphi_n-\bar{\varphi}_k|<<\pi$.

Construction of optimal polar quantizer for a given total number of clusters M can be formulated as an optimization problem:

$$E(M)=\min_{M_r,M_\varphi}\{\min_{\{R_j\}}\sum_{j=1}^{M_r}\sum_{r_n\in R_j} p_r(r_n)(r_n-\bar{r}_j)^2+$$

$$+\min_{\{\Psi_k\}}\{\sum_{k=1}^{M_\varphi}\sum_{\varphi_n\in\Psi_k} p_\varphi(\varphi_n)r_n^2(\varphi_n-\bar{\varphi}_k)^2\}\}$$

subject to: $M_rM_\varphi\leq M$.

Introducing rate-distortion functions $G_r(M_r)$ and $G_\varphi(M_\varphi)$ as quantization errors for optimal scalar quantizers on radius and phase we can formulate the optimization problem for as follows:

$$E(M)=\min_{M_r\cdot M_\varphi}\{G_r(M_r)+G_\varphi(M_\varphi)\}$$

subject to: $M_rM_\varphi\leq M$

Minimum of the cost function has to be found by optimal construction of product quantizer for polar coordinates, and optimal choice of cell numbers M_r and M_φ for radius and phase.

3.3 Dynamic programming algorithm

We attack the optimization problem under question using *Dynamic Programming* (DP) algorithm. Optimal quantizer for 1-dimensional (scalar) data can be constructed by DP algorithm [8]. The complexity of the DP algorithm is $O(MN^2)$. Wu [9] reduced the complexity of the DP algorithm to $O(MN)$, using algorithm for minimum search in monotonic matrices.

The problem of optimal product quantization for Cartesian and polar coordinates can be solved with DP algorithm as two sequential scalar quantization problems for corresponding dimensions. Cartesian coordinates (x,y) and polar coordinates (r,φ) are denoted as (u,v). Procedure consists of two steps:

Step 1: Construct two independent optimal quantizers for dimensions u and v using Dynamic Programming algorithm and calculate rate-distortion functions $G_u(m)$ and $G_v(m)$ for $m \le M$.

Step 2: Using the rate-distortion functions find optimal number of quantization cells M_u and M_v for the dimensions u and v by linear search to minimize the total error of quantization $E(M)$:

$$E(M) = \min_{1 \le M_u \le M} \{G_u(M_u) + G_v(\lceil M / M_u \rceil)\}.$$

The complexity of the first step is $O(MN)$, because it is defined by of DP algorithm for scalar quantization [8]. The complexity of the second step is at most $O(M)$. The complexity of the total algorithm is therefore $O(MN)$.

4. Vector quantization

The rejecting from rough codebook structure (like in product quantization) allows better adaptation of the codebook structure to the input data will lead to decreasing of the distortion. At the same time, the codebook will need more. We use Randomized Local Search (RLS) algorithm [1]. It is an iterative algorithm of vector quantization with property of convergence of the resulting quantizer distortion to some limit value. In general, this limit value does not depend from initial solution, but the speed of convergence does. We propose to use an optimized codebook as an initial codebook in RLS algorithm to accelerate its convergence.

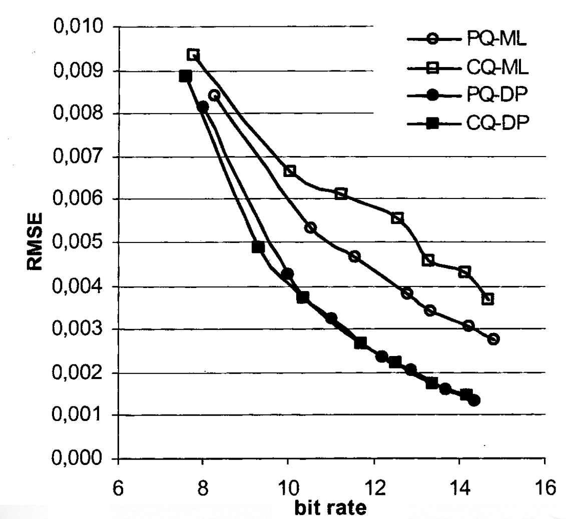

Fig 5. Rate-distortion dependency in type 1 experiments for test set #1

5. Experiments

Experimental series were taken for two test sets (see Fig. 1). The first test data set is a long digital curve with quite smooth line; the second test data set is also a single curve but with a noisy line. The points coordinates in the maps have (latitude, longitude) representation. Both maps were taken from the ESRI maps database.

The comparison was done with four different quantization approaches: Cartesian product Max-Lloyd quantizer (CQ-ML), strictly polar Max-Lloyd quantizer (PQ-ML), Cartesian product dynamic programming quantizer (CQ-DP) and strictly polar dynamic programming quantizer (PQ-DP).

We have processed two types of experiments. The first type tests were aimed at checking efficiency of the proposed algorithm in rate-distortion sense for the case when it is necessary to store the codebook in the compressed file. The second type of series was aimed at checking efficiency of the proposed method in the case when we do not need to store the codebook. In this series, we check vector quantization, where the initial codebook was generated randomly or by four predescribed methods, with the following processing in RLS algorithm (RLS, PQ-ML-RLS, CQ-ML-RLS, SD-PQ-RLS, CQ-DP-RLS).

The results of the first type of experiments series are presented in Fig 5,6,7,8. According to them the PQ-DP and CQ-DP algorithm outperforms their analogs using Max-Lloyd quantization, and have almost the same efficiency between themselves.

The results of the second type of experiments are depicted at Fig 9 and Fig 10. To estimate the increasing of convergence we processed original RLS with 100 and

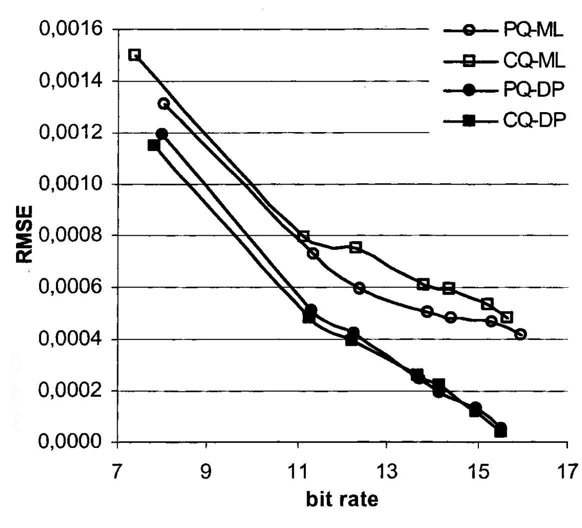

Fig 6. Rate-distortion dependency in type 1 experiments for test set #2

5000 iterations (RLS 5000). The proposed method of initializing of starting RLS codebook was processed with 100 iterations. It is shown in figures that RLS5000 slightly overrun the PQ-DP-RLS and CQ-DP-RLS algorithms for test set#1 and even looses for test set#2. In the same time PQ-DP-RLS and CQ-DP-RLS shows better results than PQ-ML-RLS and CQ-ML-RLS.

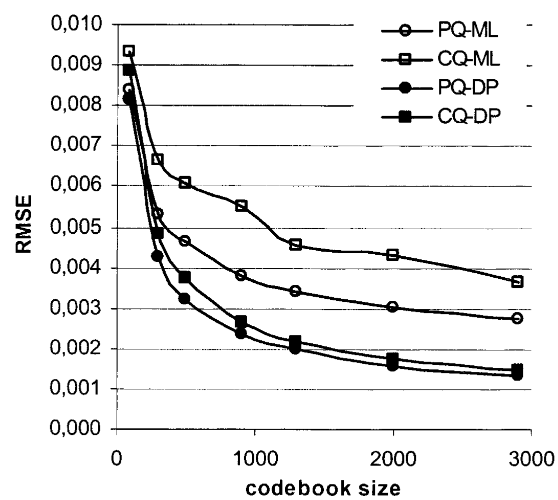

Fig. 7. The dependency between size of codebook and bit rate for test set #1

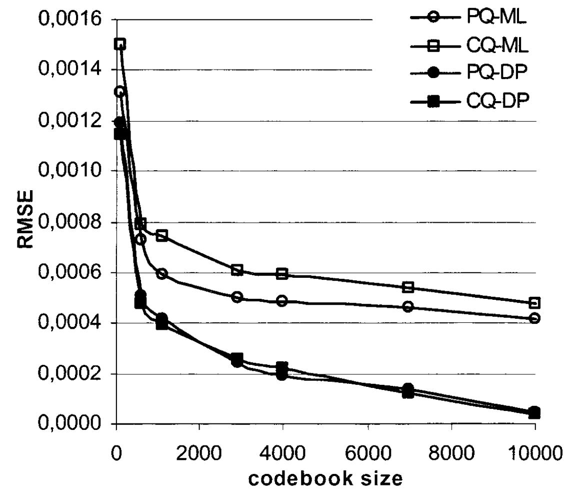

Fig. 8. The dependency between size of codebook and bit rate for test set #2

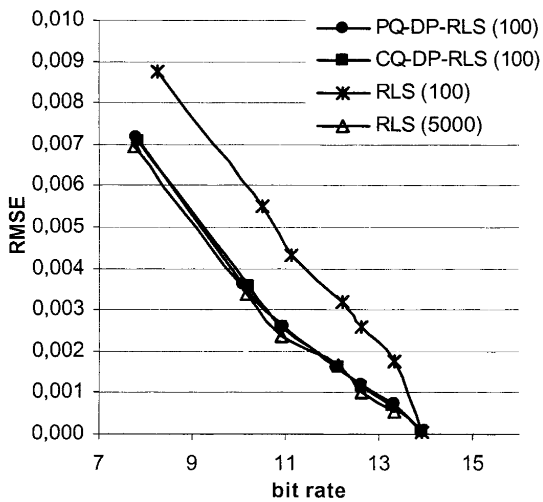

Fig 9. Rate-distortion dependency in type 2 experiments for test set #1

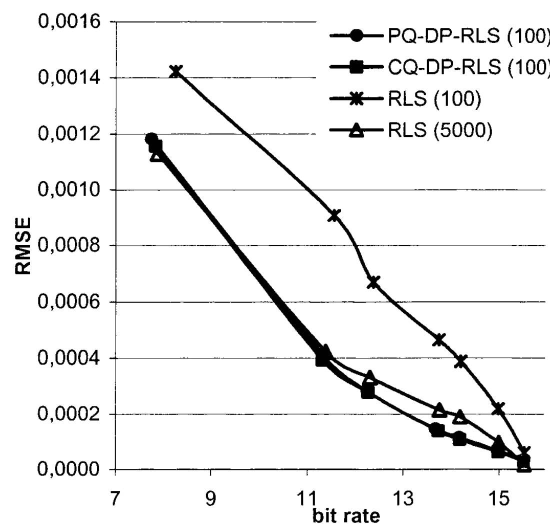

Fig 10. Rate-distortion dependency in type 2 experiments for test set #2

Max-Lloyd algorithm, used here, was processed in 200 iterations. RLS algorithm was used with two Generalized Lloyd Algorithm [10] iterations per each iteration of RLS algorithm.

6. Conclusions

The problem of quantization of vector map coordinates in vector map compression was attacked by DP algorithm as two sequential scalar quantization problems, for coordinates x and y for Cartesian quantization, and for radius r and phase φ for strictly polar quantization. This approach gives us benefit in rate-distortion sense for vector map compression problem in comparison to the use of Max-Lloyd quantization algorithm in product scalar quantization. In case of vector quantization, the usage of optimized codebook in RLS is making able to increase convergence of the algorithm.

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