



2024 IEEE International Conference on  
**Progress in Informatics and Computing**



# Modified greedy Delaunay graph-based method for TSP initialization

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22.12.2024

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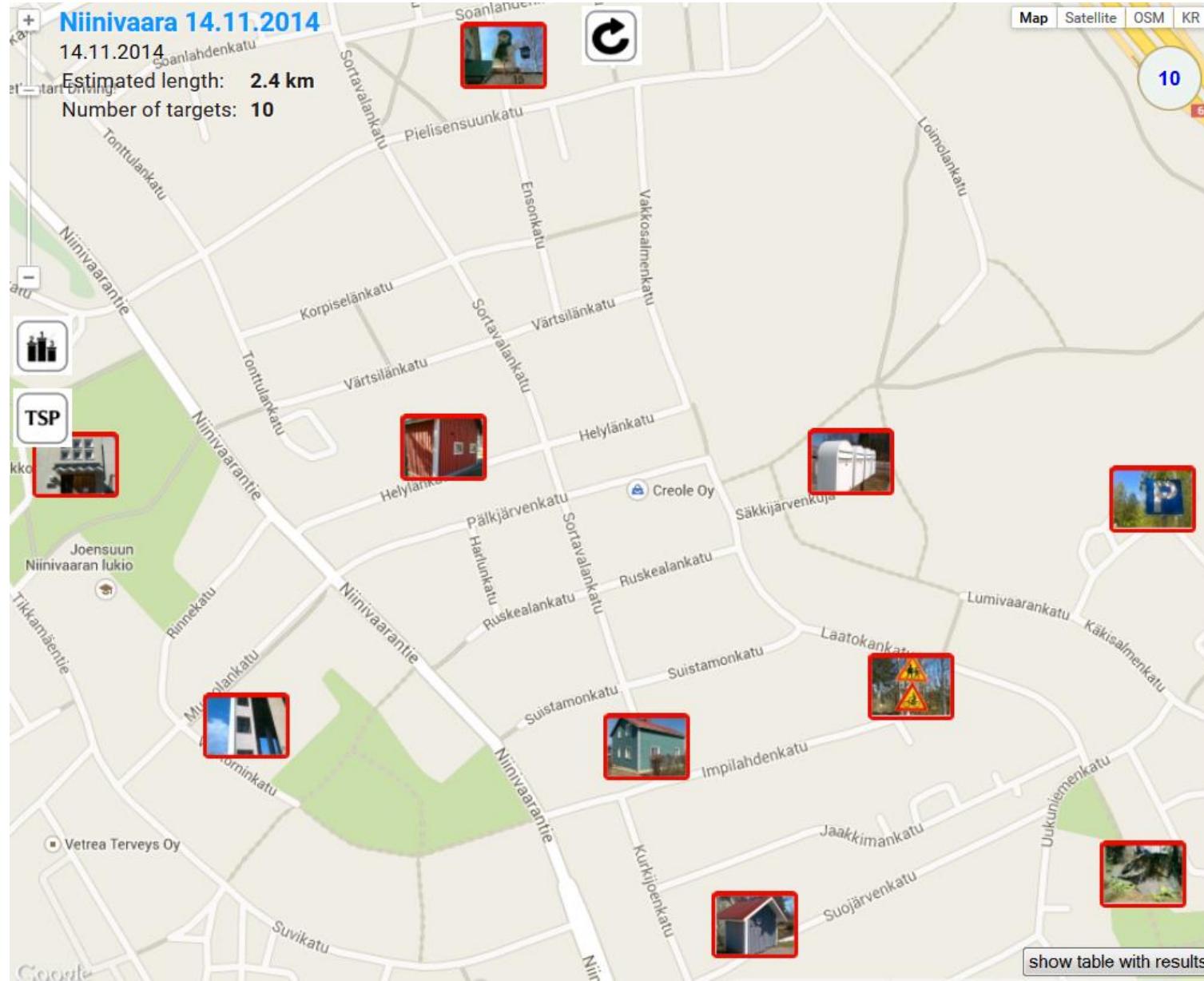


**Joensuu**

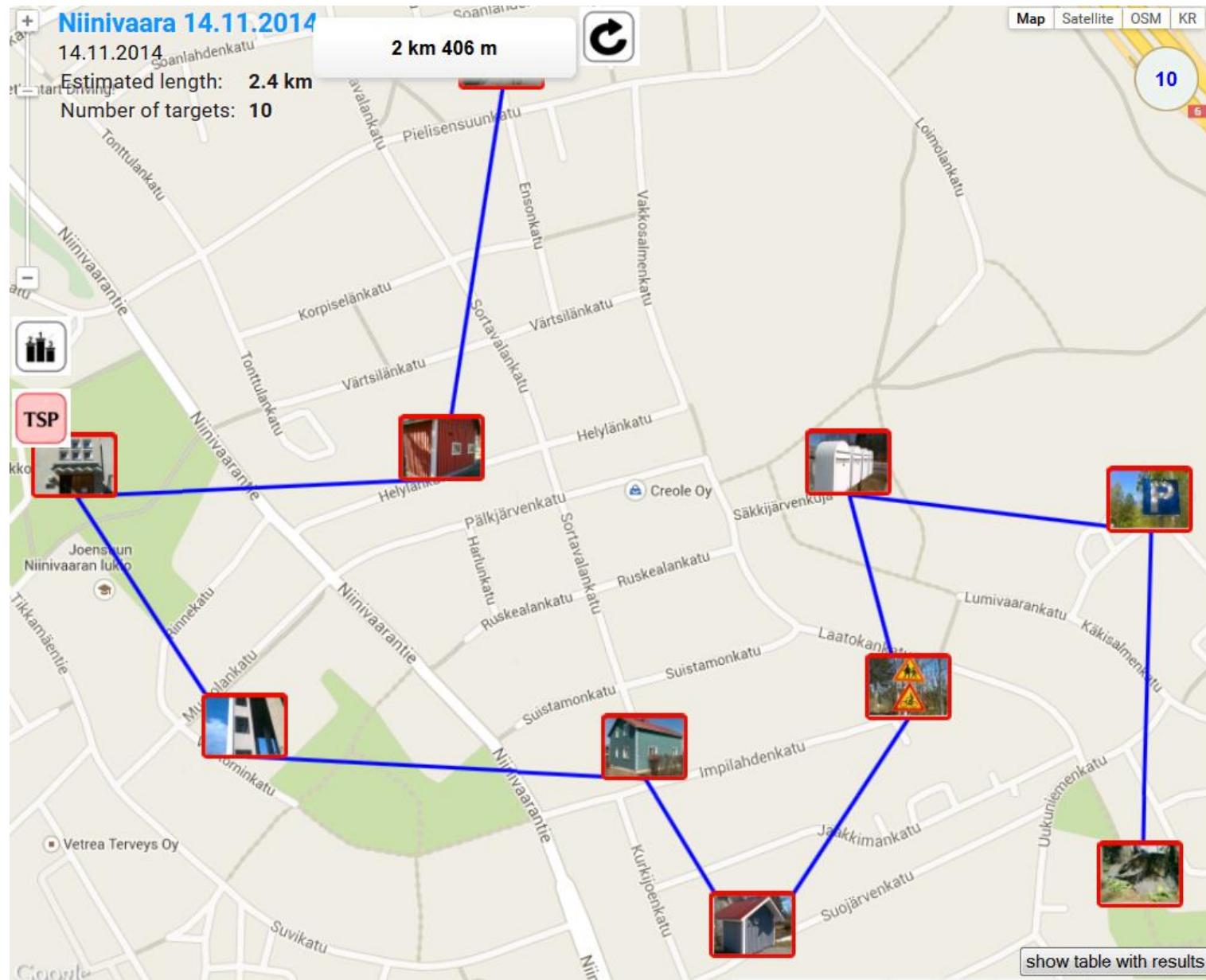


# Travelling salesman problem (TSP)

# Small-scale (open-loop) TSP

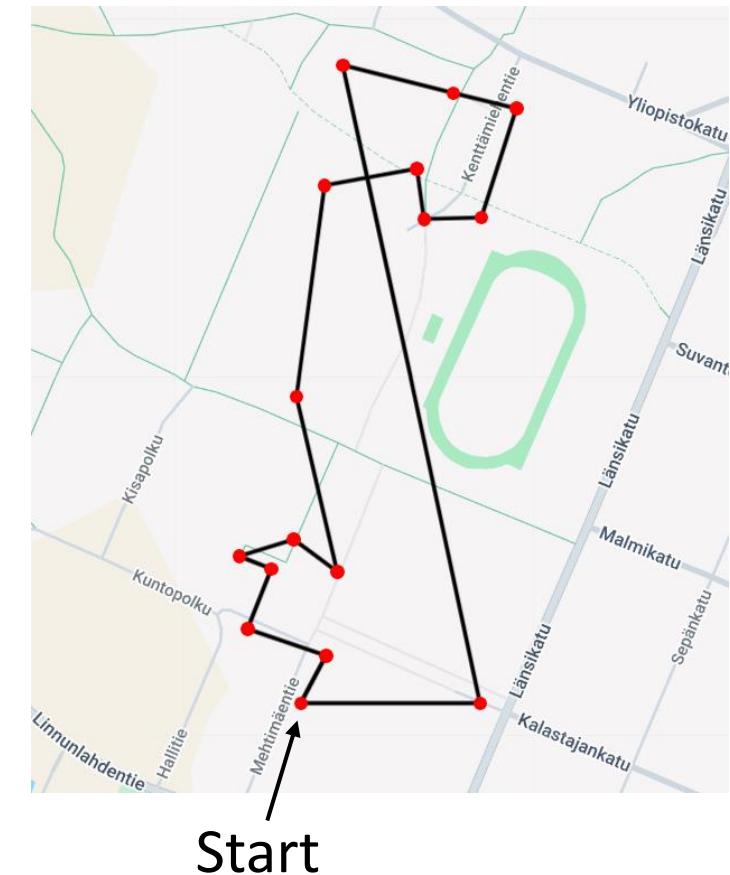


# Solution



# Simple initialization methods are slow

- Nearest neighbor approach:  
 $O(N^2)$  steps
- Too much for bigger datasets  
such as  $N = \textcolor{blue}{1 \text{ million}}$

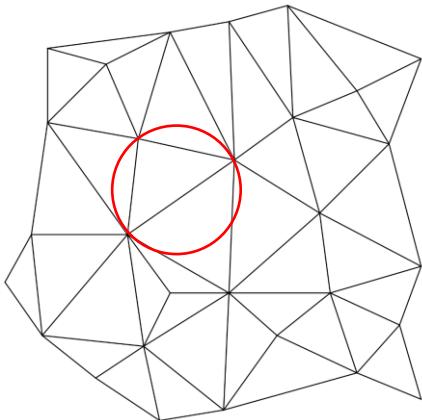


# Delaunay graph

**Dots-4345**

$n = 29$

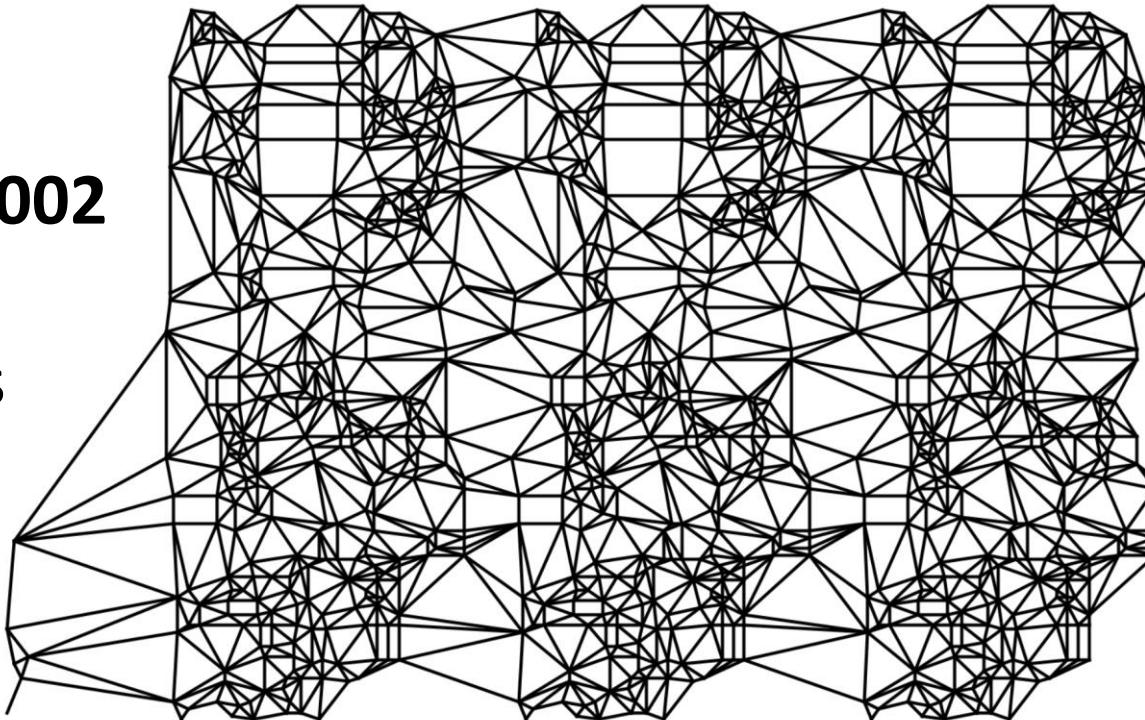
**67** edges



**TSPLIB-pr1002**

$n = 1002$

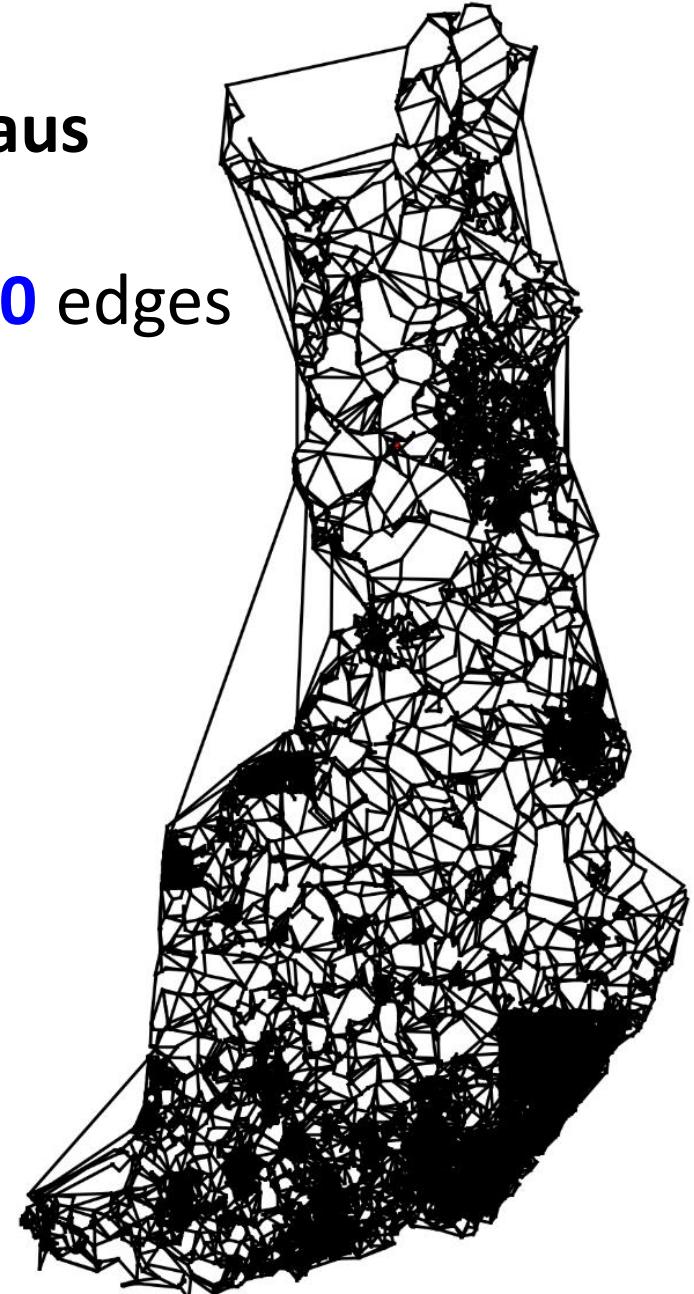
**2838** edges



**Santa Claus**

$n = 1.4M$

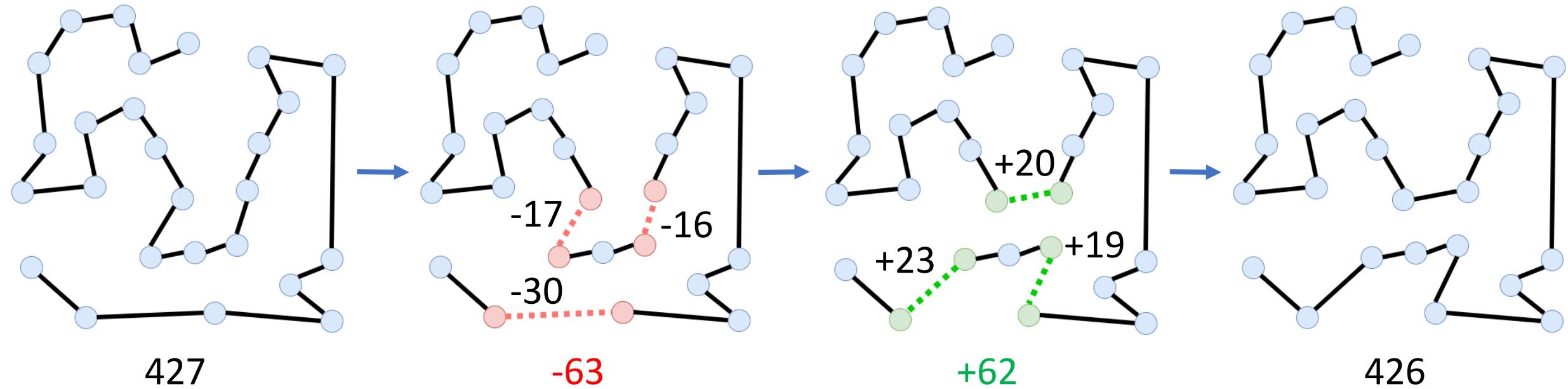
**8,623,110** edges



# State-of-the-art

- **Lin-Kernighan heuristic (LKH)**: Local search method with **k-opt** operator with localized candidate selection
- **Initialization**: Any fast heuristic
- **Good initial**: Less operations and better final solution

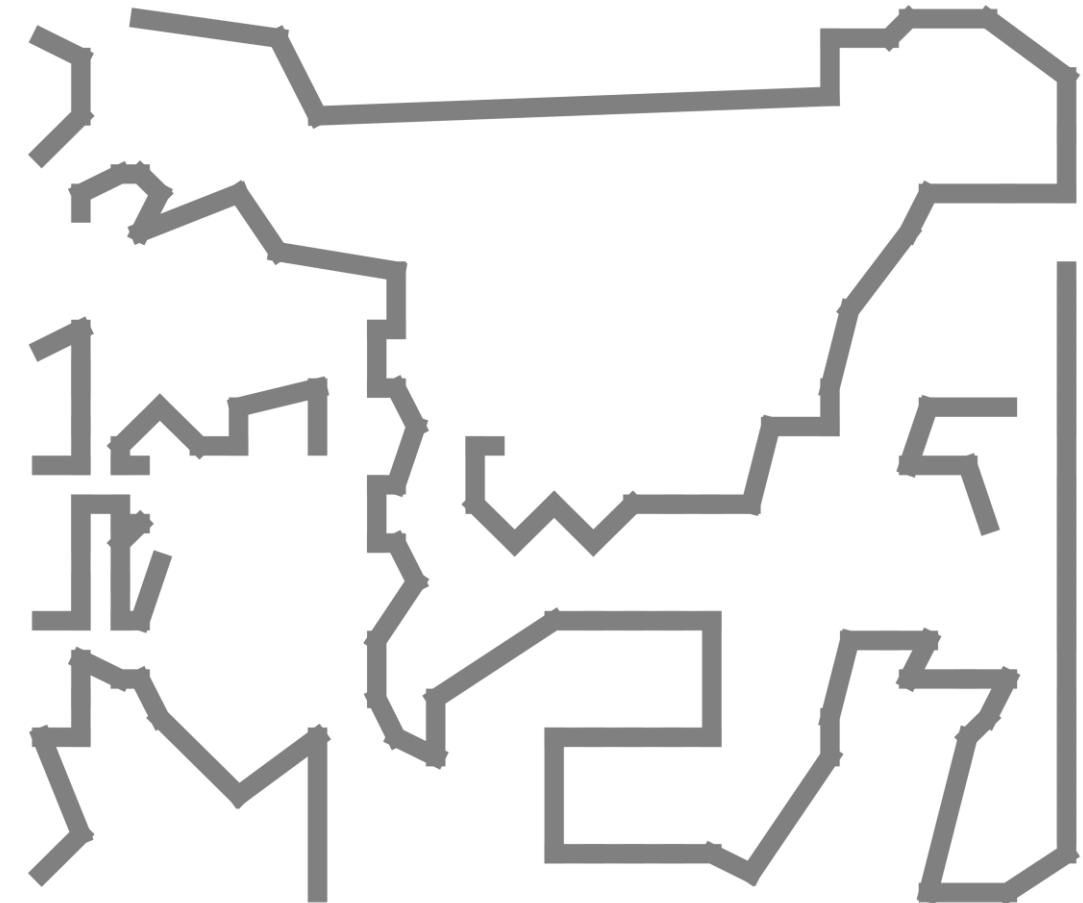
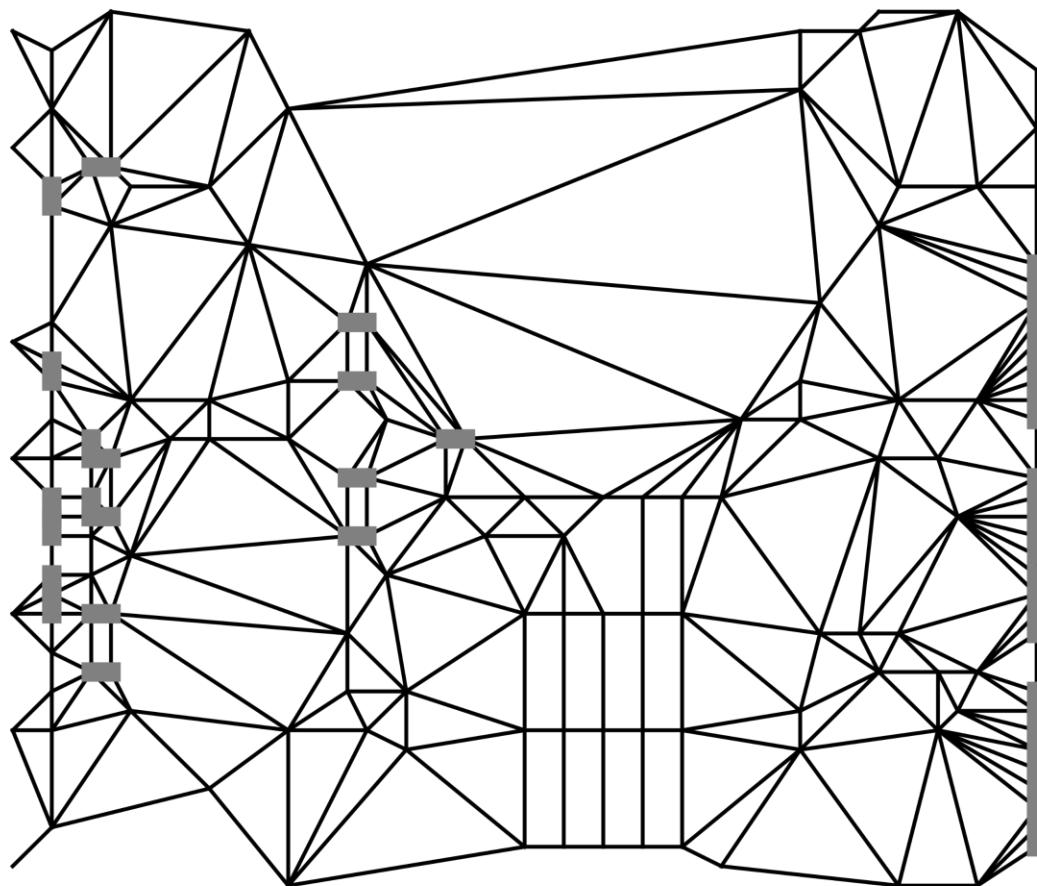
# K-opt



# Initializations considered

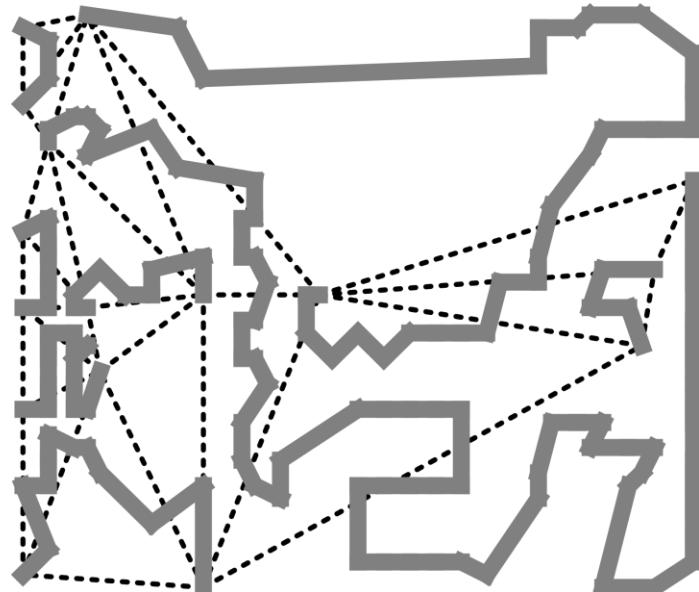
- Nearest neighbor [3, 4]
  - Nearest neighbor (fast) [2]
  - Greedy graph [2]
  - Kruskal-TSP [8]
  - Moore [9]
  - Sierpinski [10]
  - Boruvka [11]
  - Quick-Boruvka [7, 12]
  - Christofides [13, 14]
  - Walk [2]
  - Delaunay shortest edge [15]
  - Delaunay random edge [15]
  - Delaunay greedy with fragment merge (new)
- Graph-based
- Too slow or  
ineffective

# Delaunay greedy FM: 1st phase



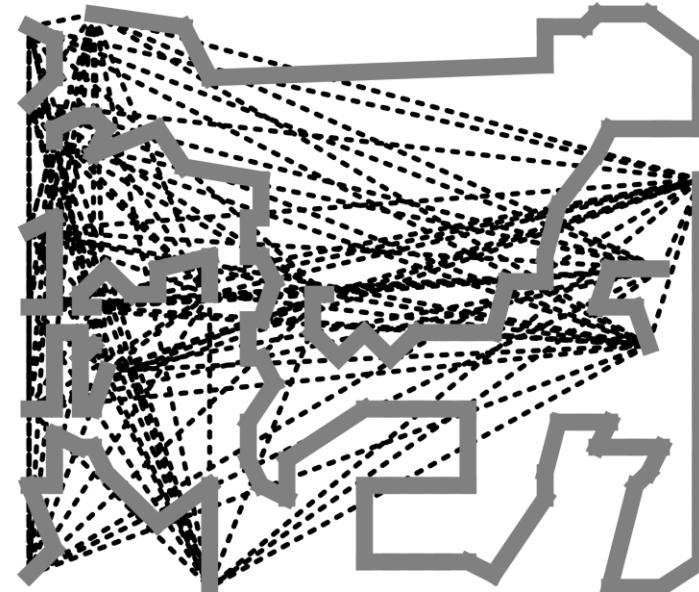
# Fragment connection methods

Re-Delaunay



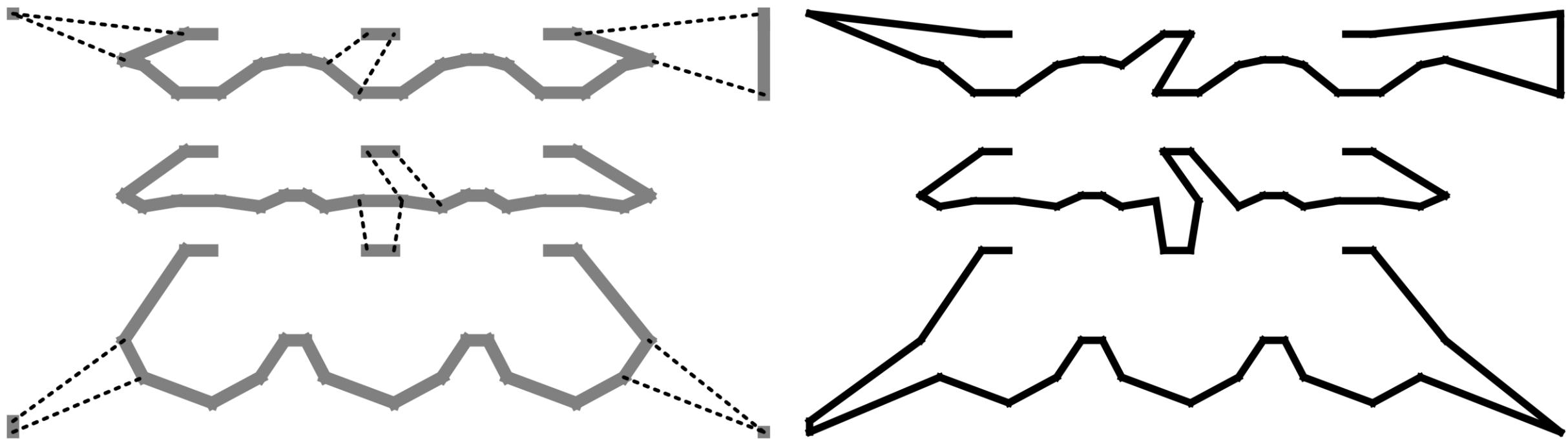
Edges 32

Full search



Edges 112

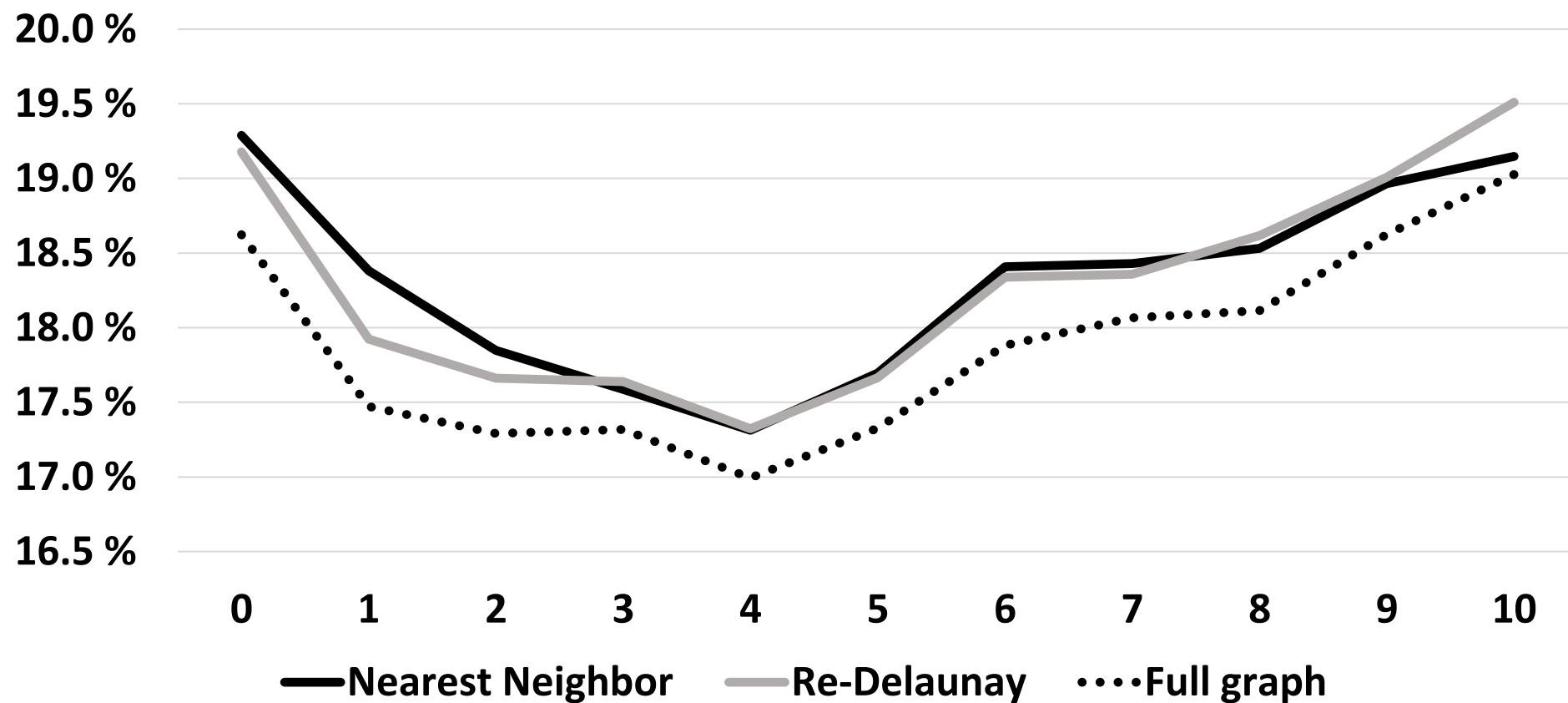
# Fragment merging



# Fragment merging values

TSPLIB results (78 instances)

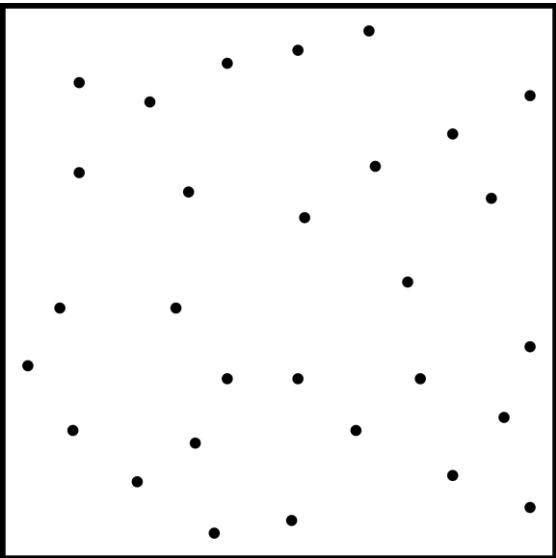
Length (gap) with different merge values



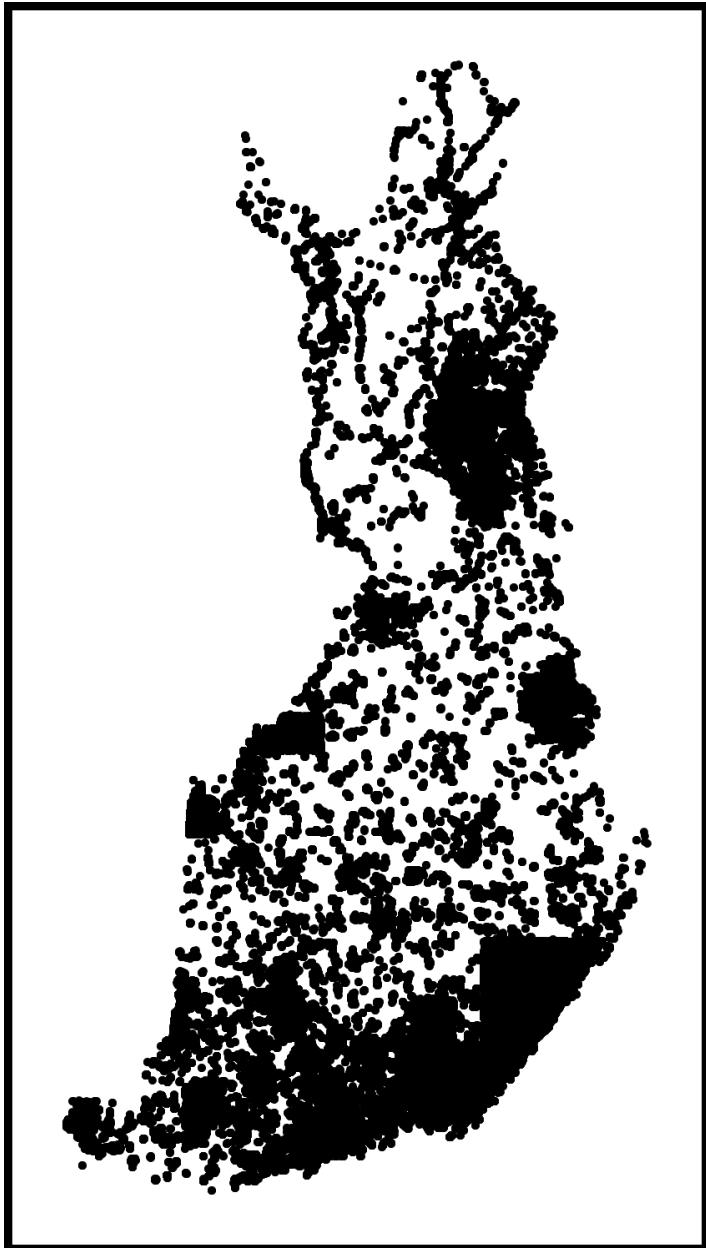
# Results

# Datasets

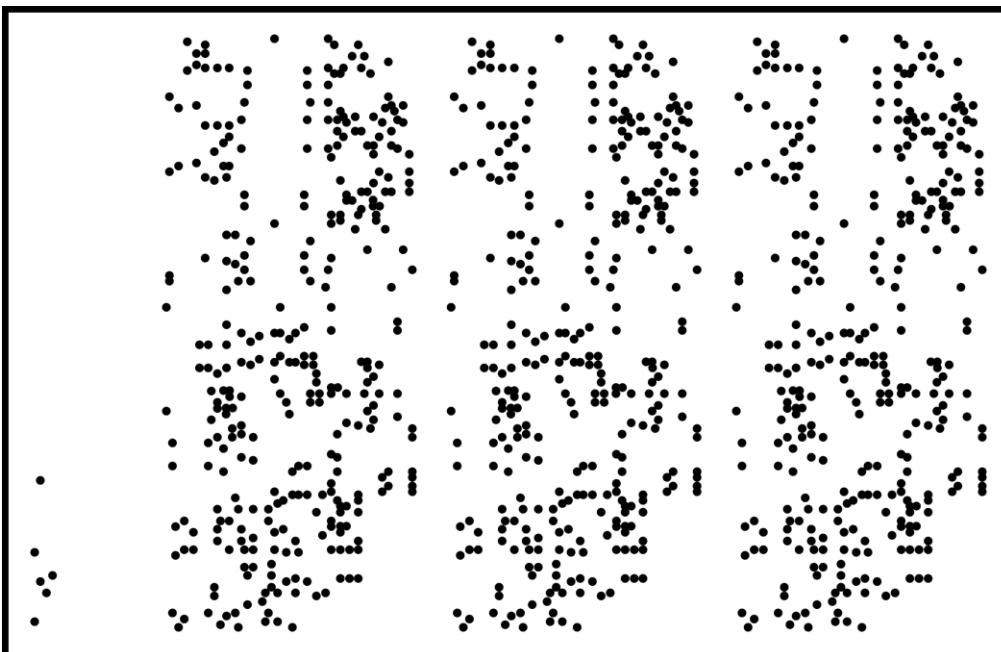
**Dots-4345**  
 $n = 29$



**Santa Claus**  
 $n = 1.4M$



**TSPLIB-pr1002**  
 $n = 1002$

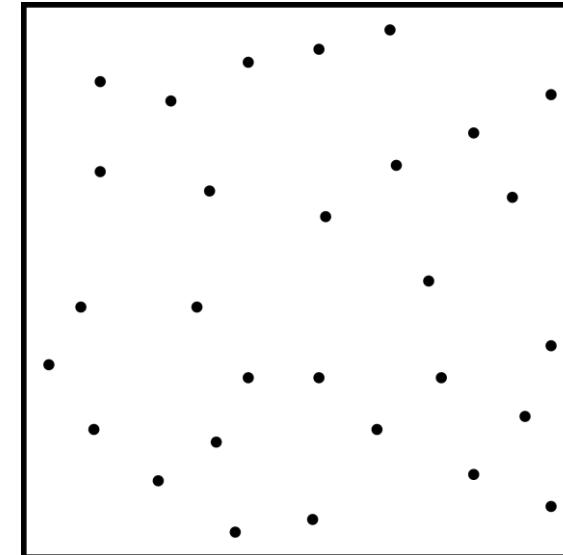


# Results on Dots

Algorithm	Average gap
NN	9.25 %
	Alpha = 0
Boruvka	21.74 %
Quick-Boruvka	14.10 %
Greedy	19.72 %
NN (fast)	24.58 %
Walk	44.82 %
Delaunay greedy FM	2.18 %
<b>LKH</b>	<b>0.32 %</b>
	Alpha = 100
Boruvka	6.53 %
Quick-Boruvka	6.69 %
Greedy	6.56 %
NN (fast)	6.74 %
Walk	14.16 %
<b>LKH</b>	<b>0.01 %</b>

**Dots-4345**

$n = 29$



6449 instances

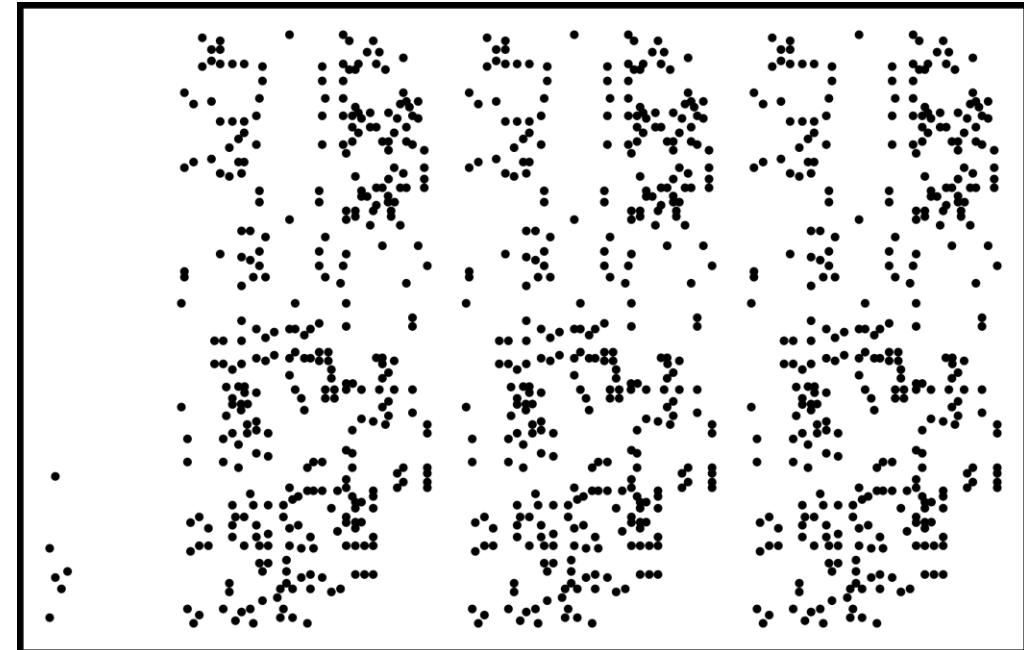
$n = 5-31$

# Results on TSPLIB

Algorithm	Average gap	Time (ms)
NN	24.08 %	71
Alpha = 0		
Boruvka	20.01 %	151
Quick-Boruvka	22.22 %	148
Greedy	19.69 %	150
NN (fast)	35.67 %	149
Walk	140.23 %	148
Delaunay greedy FM	16.99 %	25
<b>LKH</b>	<b>0.06 %</b>	<b>5440</b>
Alpha = 100		
Boruvka	12.78 %	257
Quick-Boruvka	16.17 %	255
Greedy	12.64 %	259
NN (fast)	21.29 %	256
Walk	116.17 %	258
<b>LKH</b>	<b>0.05 %</b>	<b>6370</b>

**TSPLIB-pr1002**

$n = 1002$



78 instances

$n = 51-18,512$

# Results on Santa

Algorithm	Length (km)	Time (s)
NN	-	>2h
Alpha = 0		
Boruvka	139 471	23.8
Quick-Boruvka	154 409	21.6
Greedy	132 460	23.9
NN (fast)	179 228	21.9
Walk	455 729	20.9
Delaunay greedy FM	139 270	16.6
<b>LKH</b>	<b>109 454</b>	<b>3600.0</b>
Alpha = 100		
Boruvka	128 798	495.2
Quick-Boruvka	144 222	506.0
Greedy	127 038	493.2
NN (fast)	158 508	488.1
Walk	468 314	501.1
<b>LKH</b>	<b>109 360</b>	<b>3600.0</b>



Santa Claus  
 $n = 1.4M$

# Conclusions and future work

- Simple initialization takes  $O(n^2)$  time, for Santa >2h
  - Best known: [108 417 km](#)
  - With graph: 16-20 seconds (28.5 % gap)
  - LKH optimization: 1h (0.37 % gap)
  - New result: 8 minutes (0.28 % gap)
- New Delaunay-based variant introduced with decent results
- The effect of initialization on optimization