

Please carefully read and follow the general instructions regarding exercises. Failing to meet the requirements might lead to penalties. <https://elearn.uef.fi/mod/page/view.php?id=248672>

If you suspect that something is wrong with some exercise question, please contact the lecturer.

If you face persistent issues while working on an exercise, do ask for help, e.g. during a course meeting or by contacting the lecturer via email.

**Problem 1** (Discrete wavelet transform).

a) Compute the coefficients of the discrete wavelet transform for matrix  $A$  below.

$$A = \begin{bmatrix} 0 & 3 & 6 & 8 \\ 2 & 3 & 8 & 10 \\ 4 & 3 & 8 & 12 \\ 6 & 2 & 3 & 3 \end{bmatrix}$$

**Problem 2** (Shapes to time-series). Consider the shapes in Figure 1: blue ellipse, red square and green hexadecagon.

a) Compute the centroid distance signature of each shape, doing a counter clockwise sweep from  $-\pi$  to  $\pi$  radians with  $\pi/8$  radians increments.

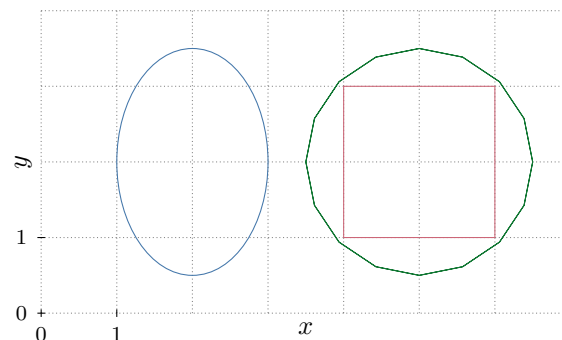


Figure 1: An ellipse, a square and an hexadecagon.

**Problem 3** (Interpolation). Consider the list of measurement stations coordinates and measured values shown in Figure 2.

a) Use inverse distance weighting over the five nearest stations to map these measurements to the  $10\text{km} \times 10\text{km}$  grid shown below, i.e. compute the interpolated values at coordinates  $(0, 0), (0, 10), (0, 20), \dots, (30, 30)$ .

b) Compute the density of measurement stations at the different grid points using kernel density estimation (KDE) with a Gaussian kernel of width 10 over the five nearest stations.

c) What information do density estimates provide with respect to the aggregated measurements?

**Problem 4** (Trajectories). Consider the trajectories shown in Figure 3.

a) Compute the dynamic time warping distance (DTW) between the two trajectories after reversing the order of the second one.

b) Apply the spatio-temporal tile transformation to convert the trajectories into sequences of symbols. Use bins of width 5 to discretize all coordinates, spatial and temporal.

	$x$	$y$	$v$
$s_1$	27	4	80
$s_2$	4	8	66
$s_3$	26	27	93
$s_4$	5	3	63
$s_5$	2	18	72
$s_6$	9	10	68
$s_7$	4	22	75
$s_8$	19	19	80
$s_9$	19	19	80
$s_{10}$	13	21	77

	$x$	$y$	$v$
$s_{11}$	26	6	78
$s_{12}$	28	15	88
$s_{13}$	15	8	69
$s_{14}$	26	22	89
$s_{15}$	5	29	80
$s_{16}$	4	22	75

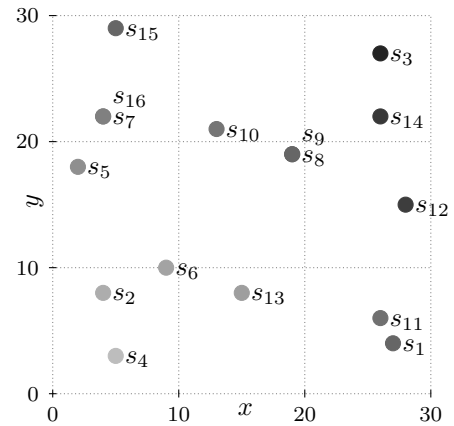


Figure 2: The list of measurement stations coordinates  $(x, y)$  and measured values  $(v)$  (left); A plot of the same measurement stations, where darker dots represent larger measured values (right).

Trajectory A

$t$	$x$	$y$
0	12.80	0.64
1	12.66	1.16
4	9.03	7.61
7	8.40	11.06
9	7.13	10.75
11	6.72	10.88
13	5.28	11.01
16	3.11	8.85

Trajectory B

$t$	$x$	$y$
0	1.45	11.27
1	2.54	9.37
4	4.78	8.92
8	4.83	8.99
9	4.85	8.98
12	7.26	10.67
15	8.95	10.37
18	9.09	9.33
21	9.12	9.33
23	9.05	8.99
27	8.97	6.56
30	9.07	5.77
33	10.76	2.88
34	13.30	0.16

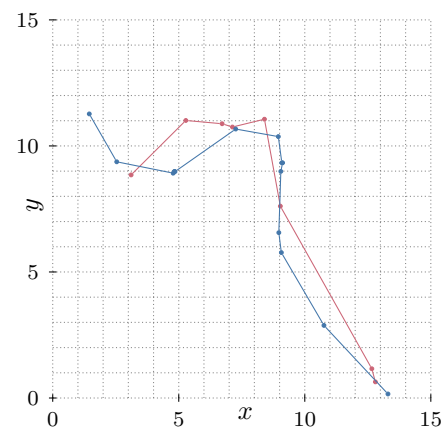


Figure 3: Two trajectories given as lists of temporal and spatial coordinates (left); A plot of the same trajectories (right).

#### PROBLEM 1

### Matrix A

0, 3, 6, 8  
2, 3, 8, 10  
4, 3, 8, 12  
6, 2, 3, 3

#### PROBLEM 3

### Station id, x, y, value

1,27,4,80  
2,4,8,66  
3,26,27,93  
4,5,3,63  
5,2,18,72  
6,9,10,68  
7,4,22,75  
8,19,19,80  
9,19,19,80  
10,13,21,77  
11,26,6,78  
12,28,15,88  
13,15,8,69  
14,26,22,89  
15,5,29,80  
16,4,22,75

#### PROBLEM 4

### t, x, y

### Trajectory A

0,12.80,0.64  
1,12.66,1.16  
4,9.03,7.61  
7,8.40,11.06  
9,7.13,10.75  
11,6.72,10.88  
13,5.28,11.01  
16,3.11,8.85

### Trajectory B

0,1.45,11.27  
1,2.54,9.37  
4,4.78,8.92  
8,4.83,8.99  
9,4.85,8.98  
12,7.26,10.67  
15,8.95,10.37  
18,9.09,9.33  
21,9.12,9.33  
23,9.05,8.99  
27,8.97,6.56  
30,9.07,5.77  
33,10.76,2.88  
34,13.30,0.16